8-2 Adding and Subtracting Rational Expressions

Find the LCM of each set of polynomials.

1. $16x, 8x^2y^3, 5x^3y$

SOLUTION:

$$\begin{aligned} 16x &= 2\cdot2\cdot2\cdot2\cdot x \\ 8x^2y^3 &= 2\cdot2\cdot2\cdot x\cdot x\cdot y\cdot y \\ 5x^3y &= 5\cdot x\cdot x\cdot x\cdot y \\ \text{LCM} &= 2\cdot2\cdot2\cdot2\cdot5\cdot x\cdot x\cdot x\cdot y\cdot y \\ &= 80x^3y^2 \end{aligned}$$

2. $7a^2, 9ab^3, 21abc^4$

SOLUTION:

$$\begin{aligned} 7a^2 &= 7\cdot a\cdot a \\ 9ab^3 &= 3\cdot3\cdot a\cdot b\cdot b\cdot b \\ 21abc^4 &= 3\cdot7\cdot a\cdot b\cdot c\cdot c\cdot c\cdot c \\ \text{LCM} &= 3\cdot3\cdot7\cdot a\cdot a\cdot b\cdot b\cdot b\cdot c\cdot c\cdot c\cdot c \\ &= 63a^2b^3c^4 \end{aligned}$$

3. $3y^2-9y, y^2-8y+15$

SOLUTION:

$$\begin{aligned} 3y^2-9y &= 3\cdot y\cdot(y-3) \\ y^2-8y+15 &= (y-5)(y-3) \\ \text{LCM} &= 3\cdot y\cdot(y-5)(y-3) \\ &= 3y(y-5)(y-3) \end{aligned}$$

4. x^3-6x^2-16x, x^2-4

SOLUTION:

$$\begin{aligned} x^3-6x^2-16x &= x\cdot(x-8)\cdot(x+2) \\ x^2-4 &= (x+2)(x-2) \\ \text{LCM} &= x\cdot(x-8)\cdot(x+2)\cdot(x-2) \\ &= x(x+2)(x-2)(x-8) \end{aligned}$$

Simplify each expression.

5. $\dfrac{12y}{5x}+\dfrac{5x}{4y^3}$

SOLUTION:

The LCD is $20xy^3$.

$$\begin{aligned} \frac{12y}{5x}+\frac{5x}{4y^3} &= \frac{12y}{5x}\cdot\frac{4y^3}{4y^3}+\frac{5x}{4y^3}\cdot\frac{5x}{5x} \\ &= \frac{48y^4+25x^2}{20xy^3} \end{aligned}$$

6. $\dfrac{5}{6ab}+\dfrac{3b^2}{14a^3}$

SOLUTION:

The LCD is $42a^3b$.

$$\begin{aligned} \frac{5}{6ab}+\frac{3b^2}{14a^3} &= \frac{5}{6ab}\cdot\frac{7a^2}{7a^2}+\frac{3b^2}{14a^3}\cdot\frac{3b}{3b} \\ &= \frac{35a^2+9b^3}{42a^3b} \end{aligned}$$

7. $\dfrac{7b}{12a}-\dfrac{1}{18ab^3}$

SOLUTION:

The LCD is $36ab^3$.

$$\begin{aligned} \frac{7b}{12a}-\frac{1}{18ab^3} &= \frac{7b}{12a}\cdot\frac{3b^3}{3b^3}-\frac{1}{18ab^3}\cdot\frac{2}{2} \\ &= \frac{21b^4-2}{36ab^3} \end{aligned}$$

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8. $\frac{y^2}{8c^2d^2} - \frac{3x}{14c^4d}$

SOLUTION:

The LCD is $56c^4d^2$.

$$\frac{y^2}{8c^2d^2} - \frac{3x}{14c^4d} = \frac{y^2}{8c^2d^2} \cdot \frac{7c^2}{7c^2} - \frac{3x}{14c^4d} \cdot \frac{4d}{4d}$$
$$= \frac{7c^2y^2 - 12dx}{56c^4d^2}$$

9. $\frac{4x}{x^2+9x+18} + \frac{5}{x+6}$

SOLUTION:

$$\frac{4x}{x^2+9x+18} + \frac{5}{x+6} = \frac{4x}{(x+3)(x+6)} + \frac{5}{x+6}$$

The LCD is $(x+3)(x+6)$.

$$\frac{4x}{x^2+9x+18} + \frac{5}{x+6} = \frac{4x}{(x+3)(x+6)} + \frac{5}{(x+6)} \cdot \frac{(x+3)}{(x+3)}$$
$$= \frac{4x+5(x+3)}{(x+3)(x+6)}$$
$$= \frac{9x+15}{(x+3)(x+6)}$$

10. $\frac{8}{y-3} + \frac{2y-5}{y^2-12y+27}$

SOLUTION:

$$\frac{8}{y-3} + \frac{2y-5}{y^2-12y+27} = \frac{8}{y-3} + \frac{2y-5}{(y-3)(y-9)}$$

The LCD is $(y-3)(y-9)$.

$$\frac{8}{y-3} + \frac{2y-5}{y^2-12y+27} = \frac{8}{y-3} \cdot \frac{(y-9)}{(y-9)} + \frac{2y-5}{(y-3)(y-9)}$$
$$= \frac{8y-72+2y-5}{(y-3)(y-9)}$$
$$= \frac{10y-77}{(y-3)(y-9)}$$

11. $\frac{4}{3x+6} - \frac{x+1}{x^2-4}$

SOLUTION:

$$\frac{4}{3x+6} - \frac{x+1}{x^2-4} = \frac{4}{3(x+2)} - \frac{x+1}{(x+2)(x-2)}$$

The LCD is $3(x+2)(x-2)$.

$$\frac{4}{3x+6} - \frac{x+1}{x^2-4} = \frac{4}{3(x+2)} \cdot \frac{(x-2)}{(x-2)} - \frac{x+1}{(x+2)(x-2)} \cdot \frac{3}{3}$$
$$= \frac{4x-8-3x-3}{3(x+2)(x-2)}$$
$$= \frac{x-11}{3(x+2)(x-2)}$$

12. $\frac{3a+2}{a^2-16} - \frac{7}{6a+24}$

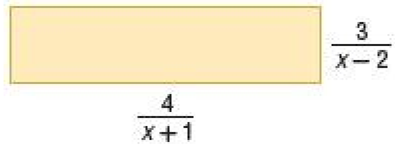
SOLUTION:

$$\frac{3a+2}{a^2-16} - \frac{7}{6a+24} = \frac{3a+2}{(a+4)(a-4)} - \frac{7}{6(a+4)}$$

The LCD is $6(a+4)(a-4)$.

$$\frac{3a+2}{a^2-16} - \frac{7}{6a+24}$$
$$= \frac{3a+2}{(a+4)(a-4)} \cdot \frac{6}{6} - \frac{7}{6(a+4)} \cdot \frac{(a-4)}{(a-4)}$$
$$= \frac{18a+12-7a+28}{6(a+4)(a-4)}$$
$$= \frac{11a+40}{6(a+4)(a-4)}$$

13. **GEOMETRY** Find the perimeter of the rectangle.



SOLUTION:

The perimeter P of the rectangle is:

$$P = 2\left(\frac{4}{x+1}\right) + 2\left(\frac{3}{x-2}\right)$$

The LCD is $(x+1)(x-2)$.

$$\begin{aligned}\frac{4}{x+1} + \frac{3}{x-2} &= \frac{8}{x+1} \cdot \frac{(x-2)}{(x-2)} + \frac{6}{x-2} \cdot \frac{(x+1)}{(x+1)} \\ &= \frac{8x-16+6x+6}{(x+1)(x-2)} \\ &= \frac{14x-10}{(x+1)(x-2)}\end{aligned}$$

Simplify each expression.

14. $\dfrac{4+\frac{2}{x}}{3-\frac{2}{x}}$

SOLUTION:

$$\begin{aligned}\frac{4+\frac{2}{x}}{3-\frac{2}{x}} &= \frac{\left(\frac{4x+2}{\cancel{x}}\right)}{\left(\frac{3x-2}{\cancel{x}}\right)} \\ &= \frac{4x+2}{3x-2}\end{aligned}$$

15. $\dfrac{6+\frac{4}{y}}{2+\frac{6}{y}}$

SOLUTION:

$$\begin{aligned}\frac{6+\frac{4}{y}}{2+\frac{6}{y}} &= \frac{\left(\frac{6y+4}{\cancel{y}}\right)}{\left(\frac{2y+6}{\cancel{y}}\right)} \\ &= \frac{6y+4}{2y+6} \\ &= \frac{3y+2}{y+3}\end{aligned}$$

16. $\dfrac{\frac{3}{x}+\frac{2}{y}}{1+\frac{4}{y}}$

SOLUTION:

$$\begin{aligned}\frac{\frac{3}{x}+\frac{2}{y}}{1+\frac{4}{y}} &= \frac{\left(\frac{3y+2x}{x\cancel{y}}\right)}{\left(\frac{y+4}{\cancel{y}}\right)} \\ &= \frac{3y+2x}{x(y+4)} \\ &= \frac{3y+2x}{xy+4x}\end{aligned}$$

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17. $\dfrac{\frac{2}{b}+\frac{5}{a}}{\frac{3}{a}-\frac{8}{b}}$

SOLUTION:

$$\dfrac{\frac{2}{b}+\frac{5}{a}}{\frac{3}{a}-\frac{8}{b}} = \dfrac{\left(\frac{2a+5b}{\cancel{ab}}\right)}{\left(\frac{3b-8a}{\cancel{ab}}\right)} = \frac{2a+5b}{3b-8a}$$

Find the LCM of each set of polynomials.

18. $24cd$, $40a^2c^3d^4$, $15abd^3$

SOLUTION:

$$\begin{aligned} 24cd &= 2\cdot2\cdot2\cdot3\cdot c\cdot d \\ 40a^2c^3d^4 &= 2\cdot2\cdot2\cdot5\cdot a\cdot a\cdot c\cdot c\cdot c\cdot d\cdot d\cdot d\cdot d \\ 15abd^3 &= 3\cdot5\cdot a\cdot b\cdot d\cdot d\cdot d \\ \text{LCM} &= 2\cdot2\cdot2\cdot3\cdot5\cdot a\cdot a\cdot b\cdot c\cdot c\cdot c\cdot d\cdot d\cdot d\cdot d \\ &= 120a^2bc^3d^4 \end{aligned}$$

19. $4x^2y^3$, $18xy^4$, $10xz^2$

SOLUTION:

$$\begin{aligned} 4x^2y^3 &= 2\cdot2\cdot x\cdot x\cdot y\cdot y\cdot y \\ 18xy^4 &= 2\cdot3\cdot3\cdot x\cdot y\cdot y\cdot y\cdot y \\ 10xz^2 &= 2\cdot5\cdot x\cdot z\cdot z \\ \text{LCM} &= 2\cdot2\cdot3\cdot3\cdot5\cdot x\cdot x\cdot y\cdot y\cdot y\cdot y\cdot z\cdot z \\ &= 180x^2y^4z^2 \end{aligned}$$

20. $x^2 - 9x + 20$, $x^2 + x - 30$

SOLUTION:

$$\begin{aligned} x^2-9x+20 &= (x-4)(x-5) \\ x^2+x-30 &= (x+6)(x-5) \\ \text{LCM} &= (x-4)(x-5)(x+6) \end{aligned}$$

21. $6x^2 + 21x - 12$, $4x^2 + 22x + 24$

SOLUTION:

$$\begin{aligned} 6x^2+21x-12 &= 3(2x^2+7x-4) \\ &= 3(x+4)(2x-1) \\ 4x^2+22x+24 &= 2(2x^2+11x+12) \\ &= 2(2x+3)(x+4) \\ \text{LCM} &= 2\cdot3\cdot(x+4)(2x-1)(2x+3) \\ &= 6(x+4)(2x-1)(2x+3) \end{aligned}$$

CCSS PERSEVERANCE Simplify each expression.

22. $\dfrac{5a}{24cf^4} + \dfrac{a}{36bc^4f^3}$

SOLUTION:

The LCD is $72bc^4f^4$.

$$\begin{aligned} &\frac{5a}{24cf^4} + \frac{a}{36bc^4f^3} \\ &= \frac{5a}{24cf^4}\cdot\frac{3bc^3}{3bc^3} + \frac{a}{36bc^4f^3}\cdot\frac{2f}{2f} \\ &= \frac{15abc^3+2af}{72bc^4f^4} \end{aligned}$$

8-2 Adding and Subtracting Rational Expressions

23. $\frac{4b}{15x^3y^2} - \frac{3b}{35x^2y^4z}$

SOLUTION:

The LCD is $105x^3y^4z$.

$$\frac{4b}{15x^3y^2} - \frac{3b}{35x^2y^4z}$$
$$= \frac{4b}{15x^3y^2} \cdot \frac{7y^2z}{7y^2z} - \frac{3b}{35x^2y^4z} \cdot \frac{3x}{3x}$$
$$= \frac{28by^2z - 9bx}{105x^3y^4z}$$

24. $\frac{5b}{6a} + \frac{3b}{10a^2} + \frac{2}{ab^2}$

SOLUTION:

The LCD is $30a^2b^2$.

$$\frac{5b}{6a} + \frac{3b}{10a^2} + \frac{2}{ab^2}$$
$$= \frac{5b}{6a} \cdot \frac{5ab^2}{5ab^2} + \frac{3b}{10a^2} \cdot \frac{3b^2}{3b^2} + \frac{2}{ab^2} \cdot \frac{30a}{30a}$$
$$= \frac{25ab^3 + 9b^3 + 60a}{30a^2b^2}$$

25. $\frac{4}{3x} + \frac{8}{x^3} + \frac{2}{5xy}$

SOLUTION:

The LCD is $15x^3y$.

$$\frac{4}{3x} + \frac{8}{x^3} + \frac{2}{5xy}$$
$$= \frac{4}{3x} \cdot \frac{5x^2y}{5x^2y} + \frac{8}{x^3} \cdot \frac{15y}{15y} + \frac{2}{5xy} \cdot \frac{3x^2}{3x^2}$$
$$= \frac{20x^2y + 120y + 6x^2}{15x^3y}$$

26. $\frac{8}{3y} + \frac{2}{9} - \frac{3}{10y^2}$

SOLUTION:

The LCD is $90y^2$.

$$\frac{8}{3y} + \frac{2}{9} - \frac{3}{10y^2}$$
$$= \frac{8}{3y} \cdot \frac{30y}{30y} + \frac{2}{9} \cdot \frac{10y^2}{10y^2} - \frac{3}{10y^2} \cdot \frac{9}{9}$$
$$= \frac{240y + 20y^2 - 27}{90y^2}$$

27. $\frac{1}{16a} + \frac{5}{12b} - \frac{9}{10b^3}$

SOLUTION:

The LCD is $240ab^3$.

$$\frac{1}{16a} + \frac{5}{12b} - \frac{9}{10b^3}$$
$$= \frac{1}{16a} \cdot \frac{15b^3}{15b^3} + \frac{5}{12b} \cdot \frac{20ab^2}{20ab^2} - \frac{9}{10b^3} \cdot \frac{24a}{24a}$$
$$= \frac{15b^3 + 100ab^2 - 216a}{240ab^3}$$

28. $\frac{8}{x^2-6x-16}+\frac{9}{x^2-3x-40}$

SOLUTION:

$$\frac{8}{x^2-6x-16}+\frac{9}{x^2-3x-40}$$
$$=\frac{8}{(x-8)(x+2)}+\frac{9}{(x-8)(x+5)}$$

The LCD is $(x-8)(x+2)(x+5)$.

$$\frac{8}{x^2-6x-16}+\frac{9}{x^2-3x-40}$$
$$=\frac{8(x+5)+9(x+2)}{(x-8)(x+2)(x+5)}$$
$$=\frac{8x+40+9x+18}{(x-8)(x+2)(x+5)}$$
$$=\frac{17x+58}{(x-8)(x+2)(x+5)}$$

29. $\frac{6}{y^2-2y-35}+\frac{4}{y^2+9y+20}$

SOLUTION:

$$\frac{6}{y^2-2y-35}+\frac{4}{y^2+9y+20}$$
$$=\frac{6}{(y-7)(y+5)}+\frac{4}{(y+5)(y+4)}$$

The LCD is $(y-7)(y+5)(y+4)$.

$$\frac{6}{y^2-2y-35}+\frac{4}{y^2+9y+20}$$
$$=\frac{6(y+4)+4(y-7)}{(y-7)(y+5)(y+4)}$$
$$=\frac{6y+24+4y-28}{(y-7)(y+5)(y+4)}$$
$$=\frac{10y-4}{(y-7)(y+5)(y+4)}$$

30. $\frac{12}{3y^2-10y-8}-\frac{3}{y^2-6y+8}$

SOLUTION:

$$\frac{12}{3y^2-10y-8}-\frac{3}{y^2-6y+8}$$
$$=\frac{12}{(y-4)(3y+2)}-\frac{3}{(y-2)(y-4)}$$

The LCD is $(y-2)(y-4)(3y+2)$.

$$\frac{12}{3y^2-10y-8}-\frac{3}{y^2-6y+8}$$
$$=\frac{12(y-2)-3(3y+2)}{(y-2)(y-4)(3y+2)}$$
$$=\frac{12y-24-9y-6}{(y-2)(y-4)(3y+2)}$$
$$=\frac{3y-30}{(y-2)(y-4)(3y+2)}$$

31. $\frac{6}{2x^2+11x-6}-\frac{8}{x^2+3x-18}$

SOLUTION:

$$\frac{6}{2x^2+11x-6}-\frac{8}{x^2+3x-18}$$
$$=\frac{6}{(2x-1)(x+6)}-\frac{8}{(x+6)(x-3)}$$

The LCD is $(2x-1)(x+6)(x-3)$.

$$\frac{6}{2x^2+11x-6}-\frac{8}{x^2+3x-18}$$
$$=\frac{6(x-3)-8(2x-1)}{(2x-1)(x+6)(x-3)}$$
$$=\frac{6x-18-16x+8}{(2x-1)(x+6)(x-3)}$$
$$=\frac{-10x-10}{(2x-1)(x+6)(x-3)}$$

32. $\frac{2x}{4x^2+9x+2}+\frac{3}{2x^2-8x-24}$

SOLUTION:

$$\frac{2x}{4x^2+9x+2}+\frac{3}{2x^2-8x-24}$$
$$=\frac{2x}{(x+2)(4x+1)}+\frac{3}{2(x+2)(x-6)}$$

The LCD is $2(x+2)(4x+1)(x-6)$.

$$\frac{2x}{4x^2+9x+2}+\frac{3}{2x^2-8x-24}$$
$$=\frac{2x[2(x-6)]+3(4x+1)}{2(x+2)(4x+1)(x-6)}$$
$$=\frac{4x^2-24x+12x+3}{2(x+2)(4x+1)(x-6)}$$
$$=\frac{4x^2-12x+3}{2(x+2)(4x+1)(x-6)}$$

33. $\frac{4x}{3x^2+3x-18}-\frac{2x}{2x^2+11x+15}$

SOLUTION:

$$\frac{4x}{3x^2+3x-18}-\frac{2x}{2x^2+11x+15}$$
$$=\frac{4x}{3(x-2)(x+3)}-\frac{2x}{(x+3)(2x+5)}$$
$$=\frac{4x(2x+5)-2x[3(x-2)]}{3(x-2)(x+3)(2x+5)}$$
$$=\frac{8x^2+20x-6x^2+12x}{3(x-2)(x+3)(2x+5)}$$
$$=\frac{2x^2+32x}{3(x-2)(x+3)(2x+5)}$$

The LCD is $3(x-2)(x+3)(2x+5)$.

34. **BIOLOGY** After a person eats something, the pH or acid level A of his or her mouth can be determined by the formula $A=\frac{20.4t}{t^2+36}+6.5,$ where t is the number of minutes that have elapsed since the food was eaten.

a. Simplify the equation.

b. What would the acid level be after 30 minutes?

SOLUTION:

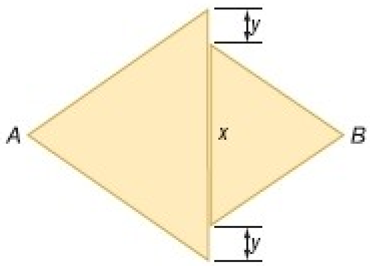
a. The LCD is t^2+36.

$$A=\frac{20.4t+6.5(t^2+36)}{t^2+36}$$
$$=\frac{20.4t+6.5t^2+234}{t^2+36}$$
$$=\frac{6.5t^2+20.4t+234}{t^2+36}$$

b. Substitute t = 30 minutes in the expression for A.

$$A==\frac{6.5(30)^2+20.4(30)+234}{(30)^2+36}$$
$$=\frac{6696}{936}$$
$$\simeq 7.2$$

35. **GEOMETRY** Both triangles in the figure at the right are equilateral. If the area of the smaller triangle is 200 square centimeters and the area of the larger triangle is 300 square centimeters, find the minimum distance from *A* to *B* in terms of *x* and *y* and simplify.



SOLUTION:

From the figure, the base length of the larger triangle is $x + 2y$ and the base length of the smaller triangle is x.

The minimum distance from *A* to *B* is the sum of the heights of the larger triangle and the smaller triangle. The height of the larger triangle is $\frac{2(300)}{x+2y}$ or $\frac{600}{x+2y}$ and the height of the smaller triangle is $\frac{2(200)}{x}$ or $\frac{400}{x}$.

So, the distance between *A* and *B* is:

$$\begin{aligned}A &= \frac{600}{x+2y}+\frac{400}{x}\\ &= \frac{600x+400(x+2y)}{x(x+2y)}\\ &= \frac{600x+400x+800y}{x(x+2y)}\\ &= \frac{1000x+800y}{x(x+2y)}\end{aligned}$$

Simplify each expression.

36. $\dfrac{\frac{2}{x-3}+\frac{3x}{x^2-9}}{\frac{3}{x+3}-\frac{4x}{x^2-9}}$

SOLUTION:

$$\begin{aligned}\frac{\frac{2}{x-3}+\frac{3x}{x^2-9}}{\frac{3}{x+3}-\frac{4x}{x^2-9}} &= \frac{\left(\frac{2}{x-3}+\frac{3x}{(x-3)(x+3)}\right)}{\left(\frac{3}{x+3}-\frac{4x}{(x-3)(x+3)}\right)}\\ &= \frac{\left(\frac{2(x+3)+3x}{\cancel{(x-3)}\cancel{(x+3)}}\right)}{\left(\frac{3(x-3)-4x}{\cancel{(x-3)}\cancel{(x+3)}}\right)}\\ &= \frac{2x+6+3x}{3x-9-4x}\\ &= \frac{5x+6}{-x-9}\end{aligned}$$

37. $\dfrac{\frac{4}{x+5}+\frac{9}{x-6}}{\frac{5}{x-6}-\frac{8}{x+5}}$

SOLUTION:

$$\begin{aligned}\frac{\frac{4}{x+5}+\frac{9}{x-6}}{\frac{5}{x-6}-\frac{8}{x+5}} &= \frac{\left(\frac{4(x-6)+9(x+5)}{\cancel{(x+5)}\cancel{(x-6)}}\right)}{\left(\frac{5(x+5)-8(x-6)}{\cancel{(x-6)}\cancel{(x+5)}}\right)}\\ &= \frac{4x-24+9x+45}{5x+25-8x+48}\\ &= \frac{13x+21}{-3x+73}\end{aligned}$$

38. $\dfrac{\dfrac{5}{x+6}-\dfrac{2x}{2x-1}}{\dfrac{x}{2x-1}+\dfrac{4}{x+6}}$

SOLUTION:

$$\begin{aligned}\dfrac{\dfrac{5}{x+6}-\dfrac{2x}{2x-1}}{\dfrac{x}{2x-1}+\dfrac{4}{x+6}}&=\dfrac{\left(\dfrac{5}{x+6}-\dfrac{2x}{2x-1}\right)}{\left(\dfrac{x}{2x-1}+\dfrac{4}{x+6}\right)}\\&=\dfrac{\left(\dfrac{5(2x-1)-2x(x+6)}{\cancel{(x+6)}\,\cancel{(2x-1)}}\right)}{\left(\dfrac{x(x+6)+4(2x-1)}{\cancel{(2x-1)}\,\cancel{(x+6)}}\right)}\\&=\dfrac{10x-5-2x^2-12x}{x^2+6x+8x-4}\\&=\dfrac{-2x^2-2x-5}{x^2+14x-4}\end{aligned}$$

39. $\dfrac{\dfrac{8}{x-9}-\dfrac{x}{3x+2}}{\dfrac{3}{3x+2}+\dfrac{4x}{x-9}}$

SOLUTION:

$$\begin{aligned}\dfrac{\dfrac{8}{x-9}-\dfrac{x}{3x+2}}{\dfrac{3}{3x+2}+\dfrac{4x}{x-9}}&=\dfrac{\left(\dfrac{8(3x+2)-x(x-9)}{\cancel{(x-9)}\,\cancel{(3x+2)}}\right)}{\left(\dfrac{3(x-9)+4x(3x+2)}{\cancel{(3x+2)}\,\cancel{(x-9)}}\right)}\\&=\dfrac{24x+16-x^2+9x}{3x-27+12x^2+8x}\\&=\dfrac{-x^2+33x+16}{12x^2+11x-27}\end{aligned}$$

40. **OIL PRODUCTION** Managers of an oil company have estimated that oil will be pumped from a certain well at a rate based on the function $R(x)=\dfrac{20}{x}+\dfrac{200x}{3x^2+20}$, where $R(x)$ is the rate of production in thousands of barrels per year x years after pumping begins.

a. Simplify $R(x)$.

b. At what rate will oil be pumping from the well in 50 years?

SOLUTION:

a.

$$\begin{aligned}R(x)&=\dfrac{20}{x}+\dfrac{200x}{3x^2+20}\\&=\dfrac{20(3x^2+20)+200x(x)}{x(3x^2+20)}\\&=\dfrac{60x^2+400+200x^2}{3x^3+20x}\\&=\dfrac{260x^2+400}{3x^3+20x}\end{aligned}$$

b. Substitute $x = 50$ in $R(x)$.

$$\begin{aligned}R(50)&=\dfrac{260(50)^2+400}{3(50)^3+20(50)}\\&=\dfrac{260(2500)+400}{3(125000)+1000}\\&=\dfrac{650000+400}{375000+1000}\\&=\dfrac{650400}{376000}\\&\approx 1.73\end{aligned}$$

Therefore, the rate of oil pumping from the well in 50 years is about 1730 barrels per year.

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Find the LCM of each set of polynomials.

41. $12xy^4$, $14x^4y^2$, $5xyz^3$, $15x^5y^3$

SOLUTION:

$$\begin{aligned} 12xy^4 &= 2\cdot 2\cdot 3\cdot x\cdot y\cdot y\cdot y\cdot y \\ 14x^4y^2 &= 2\cdot 7\cdot x\cdot x\cdot x\cdot x\cdot y\cdot y \\ 5xyz^3 &= 5\cdot x\cdot y\cdot z\cdot z\cdot z \\ 15x^5y^3 &= 3\cdot 5\cdot x\cdot x\cdot x\cdot x\cdot x\cdot y\cdot y\cdot y \\ \text{LCM} &= 2\cdot 2\cdot 3\cdot 5\cdot 7\cdot x\cdot x\cdot x\cdot x\cdot x\cdot y\cdot y\cdot y\cdot y\cdot z\cdot z\cdot z \\ &= 420x^5y^4z^3 \end{aligned}$$

42. $-6abc^2$, $18a^2b^2$, $15a^4c$, $8b^3$

SOLUTION:

$$\begin{aligned} -6abc^2 &= -2\cdot 3\cdot a\cdot b\cdot c\cdot c \\ 18a^2b^2 &= 2\cdot 3\cdot 3\cdot a\cdot a\cdot b\cdot b \\ 15a^4c &= 3\cdot 5\cdot a\cdot a\cdot a\cdot a\cdot c \\ 8b^3 &= 2\cdot 2\cdot 2\cdot b\cdot b\cdot b \\ \text{LCM} &= -2\cdot 2\cdot 2\cdot 3\cdot 3\cdot 5\cdot a\cdot a\cdot a\cdot a\cdot b\cdot b\cdot b\cdot c\cdot c \\ &= -360a^4b^3c^2 \end{aligned}$$

43. $x^2 - 3x - 28$, $2x^2 + 9x + 4$, $x^2 - 16$

SOLUTION:

$$\begin{aligned} x^2 - 3x - 28 &= (x-7)(x+4) \\ 2x^2 + 9x + 4 &= (x+4)(2x+1) \\ x^2 - 16 &= (x+4)(x-4) \\ \text{LCM} &= (x-7)(x+4)(x-4)(2x+1) \end{aligned}$$

44. $x^2 - 5x - 24$, $x^2 - 9$, $3x^2 + 8x - 3$

SOLUTION:

$$\begin{aligned} x^2 - 5x - 24 &= (x-8)(x+3) \\ x^2 - 9 &= (x+3)(x-3) \\ 3x^2 + 8x - 3 &= (x+3)(3x-1) \\ \text{LCM} &= (x-8)(x+3)(x-3)(3x-1) \end{aligned}$$

Simplify each expression.

45. $\frac{1}{12a} + 6 - \frac{3}{5a^2}$

SOLUTION:

The LCD is $60a^2$.

$$\begin{aligned} \frac{1}{12a} + 6 - \frac{3}{5a^2} &= \frac{1}{12a}\cdot\frac{5a}{5a} + 6\cdot\frac{60a^2}{60a^2} - \frac{3}{5a^2}\cdot\frac{12}{12} \\ &= \frac{5a + 360a^2 - 36}{60a^2} \\ &= \frac{360a^2 + 5a - 36}{60a^2} \end{aligned}$$

46. $\frac{5}{16y^2} - 4 - \frac{8}{3x^2y}$

SOLUTION:

The LCD is $48x^2y^2$.

$$\frac{5}{16y^2} - 4 - \frac{8}{3x^2y}$$

$$= \frac{5}{16y^2}\cdot\frac{3x^2}{3x^2} - 4\cdot\frac{48x^2y^2}{48x^2y^2} - \frac{8}{3x^2y}\cdot\frac{16y}{16y}$$

$$= \frac{15x^2 - 192x^2y^2 - 128y}{48x^2y^2}$$

47. $\frac{5}{6x^2+46x-16}+\frac{2}{6x^2+57x+72}$

SOLUTION:

$$\frac{5}{6x^2+46x-16}+\frac{2}{6x^2+57x+72}$$
$$=\frac{5}{2(x+8)(3x-1)}+\frac{2}{3(2x+3)(x+8)}$$

The LCD is $6(x+8)(3x-1)(2x+3)$.

$$\frac{5}{6x^2+46x-16}+\frac{2}{6x^2+57x+72}$$
$$=\frac{5\cdot3(2x+3)+2\cdot2(3x-1)}{6(x+8)(3x-1)(2x+3)}$$
$$=\frac{30x+45+12x-4}{6(x+8)(3x-1)(2x+3)}$$
$$=\frac{42x+41}{6(x+8)(3x-1)(2x+3)}$$

48. $\frac{1}{8x^2-20x-12}+\frac{4}{6x^2+27x+12}$

SOLUTION:

$$\frac{1}{8x^2-20x-12}+\frac{4}{6x^2+27x+12}$$
$$=\frac{1}{4(2x+1)(x-3)}+\frac{4}{3(x+4)(2x+1)}$$

The LCD is $12(2x+1)(x-3)(x+4)$.

$$\frac{1}{8x^2-20x-12}+\frac{4}{6x^2+27x+12}$$
$$=\frac{1\cdot3(x+4)+4\cdot4(x-3)}{12(2x+1)(x-3)(x+4)}$$
$$=\frac{3x+12+16x-48}{12(2x+1)(x-3)(x+4)}$$
$$=\frac{19x-36}{12(2x+1)(x-3)(x+4)}$$

49. $\frac{x^2+y^2}{x^2-y^2}+\frac{y}{x+y}-\frac{x}{x-y}$

SOLUTION:

$$\frac{x^2+y^2}{x^2-y^2}+\frac{y}{x+y}-\frac{x}{x-y}$$
$$=\frac{x^2+y^2}{(x-y)(x+y)}+\frac{y}{x+y}-\frac{x}{x-y}$$

The LCD is $(x-y)(x+y)$.

$$\frac{x^2+y^2}{x^2-y^2}+\frac{y}{x+y}-\frac{x}{x-y}$$
$$=\frac{x^2+y^2+y(x-y)-x(x+y)}{(x-y)(x+y)}$$
$$=\frac{x^2+y^2+yx-y^2-x^2-xy}{(x-y)(x+y)}$$
$$=0$$

50. $\frac{x^2+x}{x^2-9x+8}+\frac{4}{x-1}-\frac{3}{x-8}$

SOLUTION:

$$\frac{x^2+x}{x^2-9x+8}+\frac{4}{x-1}-\frac{3}{x-8}$$
$$=\frac{x^2+x}{(x-8)(x-1)}+\frac{4}{x-1}-\frac{3}{x-8}$$

The LCD is $(x-8)(x-1)$.

$$\frac{x^2+x}{x^2-9x+8}+\frac{4}{x-1}-\frac{3}{x-8}$$
$$=\frac{x^2+x+4(x-8)-3(x-1)}{(x-8)(x-1)}$$
$$=\frac{x^2+x+4x-32-3x+3}{(x-8)(x-1)}$$
$$=\frac{x^2+2x-29}{x^2-9x+8}$$

51. $\dfrac{\dfrac{2}{a-1}+\dfrac{3}{a-4}}{\dfrac{6}{a^2-5a+4}}$

SOLUTION:

$$\begin{aligned}\dfrac{\dfrac{2}{a-1}+\dfrac{3}{a-4}}{\dfrac{6}{a^2-5a+4}} &= \dfrac{\dfrac{2(a-4)+3(a-1)}{\cancel{(a-1)}\,\cancel{(a-4)}}}{\dfrac{6}{\cancel{(a-1)}\,\cancel{(a-4)}}}\\ &= \dfrac{2a-8+3a-3}{6}\\ &= \dfrac{5a-11}{6}\end{aligned}$$

52. $\dfrac{\dfrac{1}{x}+\dfrac{1}{y}}{\left(\dfrac{1}{x}-\dfrac{1}{y}\right)(x+y)}$

SOLUTION:

$$\begin{aligned}\dfrac{\dfrac{1}{x}+\dfrac{1}{y}}{\left(\dfrac{1}{x}-\dfrac{1}{y}\right)(x+y)} &= \dfrac{\dfrac{\cancel{y+x}}{\cancel{xy}}}{\left(\dfrac{y-x}{\cancel{xy}}\right)\cancel{(x+y)}}\\ &= \dfrac{1}{y-x}\end{aligned}$$

53. **GEOMETRY** An expression for the length of one rectangle is $\dfrac{x^2-9}{x-2}$. The length of a similar rectangle is expressed as $\dfrac{x+3}{x^2-4}$. What is the scale factor of the two rectangles? Write in simplest form.

SOLUTION:

Divide the expressions to find the scale factor.

$$\begin{aligned}\text{The scale factor of the two triangles} &= \dfrac{\left(\dfrac{x^2-9}{x-2}\right)}{\left(\dfrac{x+3}{x^2-4}\right)}\\ &= \dfrac{\left(\dfrac{\cancel{(x+3)}(x-3)}{\cancel{x-2}}\right)}{\left(\dfrac{\cancel{x+3}}{(x+2)\cancel{(x-2)}}\right)}\\ &= (x-3)(x+2)\end{aligned}$$

That is, the scale factor is $(x-3)(x+2)$ to 1.

54. **CCSS MODELING** Cameron is taking a 20-mile kayaking trip. He travels half the distance at one rate. The rest of the distance he travels 2 miles per hour slower.

a. If *x* represents the faster pace in miles per hour, write an expression that represents the time spent at that pace.

b. Write an expression for the amount of time spent at the slower pace.

c. Write an expression for the amount of time Cameron needed to complete the trip.

SOLUTION:

a. $\frac{10}{x}$

b. $\frac{10}{x-2}$

c.

$$\begin{aligned}\frac{10}{x}+\frac{10}{x-2}&=\frac{10(x-2)+10x}{x(x-2)}\\&=\frac{10x-20+10x}{x(x-2)}\\&=\frac{20x-20}{x(x-2)}\\&=\frac{20(x-1)}{x(x-2)}\end{aligned}$$

Find the slope of the line that passes through each pair of points.

55. $A\left(\frac{2}{p},\frac{1}{2}\right)$ and $B\left(\frac{1}{3},\frac{3}{p}\right)$

SOLUTION:

The slope of the line AB is:

$$\begin{aligned}m&=\frac{\left(\frac{3}{p}-\frac{1}{2}\right)}{\left(\frac{1}{3}-\frac{2}{p}\right)}\\&=\frac{\left(\frac{6-p}{2p}\right)}{\left(\frac{p-6}{3p}\right)}\\&=-\frac{3}{2}\end{aligned}$$

56. $C\left(\frac{1}{4},\frac{4}{q}\right)$ and $D\left(\frac{5}{q},\frac{1}{5}\right)$

SOLUTION:

The slope of the line CD is:

$$\begin{aligned}m&=\frac{\left(\frac{1}{5}-\frac{4}{q}\right)}{\left(\frac{5}{q}-\frac{1}{4}\right)}\\&=\frac{\left(\frac{q-20}{5q}\right)}{\left(\frac{20-q}{4q}\right)}\\&=-\frac{4}{5}\end{aligned}$$

57. $E\left(\frac{7}{w},\frac{1}{7}\right)$ and $F\left(\frac{1}{7},\frac{7}{w}\right)$

SOLUTION:

The slope of the line EF is:

$$\begin{aligned} m &= \frac{\left(\frac{7}{w}-\frac{1}{7}\right)}{\left(\frac{1}{7}-\frac{7}{w}\right)} \\ &= \frac{\left(\frac{49-w}{7w}\right)}{\left(\frac{w-49}{7w}\right)} \\ &= -1 \end{aligned}$$

58. $G\left(\frac{6}{n},\frac{1}{6}\right)$ and $H\left(\frac{1}{6},\frac{6}{n}\right)$

SOLUTION:

The slope of the line GH is:

$$\begin{aligned} m &= \frac{\left(\frac{6}{n}-\frac{1}{6}\right)}{\left(\frac{1}{6}-\frac{6}{n}\right)} \\ &= \frac{\left(\frac{36-n}{6n}\right)}{\left(\frac{n-36}{6n}\right)} \\ &= -1 \end{aligned}$$

59. **PHOTOGRAPHY** The focal length of a lens establishes the field of view of the camera. The shorter the focal length is, the larger the field of view. For a camera with a fixed focal length of 70 mm to focus on an object x mm from the lens, the film must be placed a distance y from the lens. This is represented by $\frac{1}{x}+\frac{1}{y}=\frac{1}{70}$.

a. Express y as a function of x.

b. What happens to the focusing distance when the object is 70 mm away?

SOLUTION:

a.

$$\begin{aligned} \frac{1}{x}+\frac{1}{y} &= \frac{1}{70} \\ \frac{1}{y} &= \frac{1}{70}-\frac{1}{x} \\ \frac{1}{y} &= \frac{x-70}{70x} \\ y &= \frac{70x}{x-70} \end{aligned}$$

b. Sample answer: When the object is 70 mm away, y needs to be 0, which is impossible.

60. **PHARMACOLOGY** Two drugs are administered to a patient. The concentrations in the bloodstream of each are given by $f(t) = \dfrac{2t}{3t^2 + 9t + 6}$ and $g(t) = \dfrac{3t}{2t^2 + 6t + 4}$ where t is the time, in hours, after the drugs are administered.

a. Add the two functions together to determine a function for the total concentration of drugs in the patient's bloodstream.

b. What is the concentration of drugs after 8 hours?

SOLUTION:

a. The total concentration of drugs in the patient's bloodstream is:

$$\begin{aligned} h(t) &= \frac{2t}{3t^2 + 9t + 6} + \frac{3t}{2t^2 + 6t + 4} \\ &= \frac{2t}{3(t^2 + 3t + 2)} + \frac{3t}{2(t^2 + 3t + 2)} \\ &= \frac{4t + 9t}{6(t^2 + 3t + 2)} \\ &= \frac{13t}{6t^2 + 18t + 12} \end{aligned}$$

b. Substitute $t = 8$ in the expression for $h(t)$. The concentration of drugs after 8 hours is:

$$\begin{aligned} h(8) &= \frac{13(8)}{6(8)^2 + 18(8) + 12} \\ &\approx 0.19 \end{aligned}$$

61. **DOPPLER EFFECT** Refer to the application at the beginning of the lesson. George is equidistant from two fire engines traveling toward him from opposite directions.

a. Let x be the speed of the faster fire engine and y be the speed of the slower fire engine. Write and simplify a rational expression representing the difference in pitch between the two sirens according to George.

b. If one is traveling at 45 meters per second and the other is traveling at 70 meters per second, what is the difference in their pitches according to George? The speed of sound in air is 332 meters per second, and both engines have a siren with a pitch of 500 Hz.

SOLUTION:

a. Doppler effect of the faster fire engine is represented by the rational expression $\dfrac{P_0S_0}{(S_0 - x)}$.

Doppler effect of the slower fire engine is represented by the rational expression $\dfrac{P_0S_0}{(S_0 - y)}$.

$$\begin{aligned} \frac{P_0S_0}{(S_0 - x)} - \frac{P_0S_0}{(S_0 - y)} &= \frac{P_0S_0(S_0 - y) - P_0S_0(S_0 - x)}{(S_0 - x)(S_0 - y)} \\ &= \frac{P_0S_0[S_0 - y - S_0 + x]}{(S_0 - x)(S_0 - y)} \\ &= \frac{P_0S_0(x - y)}{(S_0 - x)(S_0 - y)} \\ &= \frac{P_0S_0x - P_0S_0y}{(S_0 - x)(S_0 - y)} \end{aligned}$$

b. Substitute $x = 70$, $y = 45$, $S_0 = 332$, and $P_0 = 500$ in the expression $\dfrac{P_0S_0x - P_0S_0y}{(S_0 - x)(S_0 - y)}$.

$$\begin{aligned} \frac{P_0S_0x - P_0S_0y}{(S_0 - x)(S_0 - y)} &= \frac{(500)(332)(70) - (500)(332)(45)}{(332 - 70)(332 - 45)} \\ &= \frac{11620000 - 7470000}{(262)(287)} \\ &= \frac{4150000}{75194} \\ &\approx 55.2 \end{aligned}$$

According to George, the difference in their pitches is about 55.2 Hz.

62. **RESEARCH** A student studying learning behavior performed an experiment in which a rat was repeatedly sent through a maze. It was determined that the time it took the rat to complete the maze followed the rational function $T(x) = 4 + \frac{10}{x}$, where x represented the number of trials.

a. What is the domain of the function?

b. Graph the function for $0 \le x \le 10$.

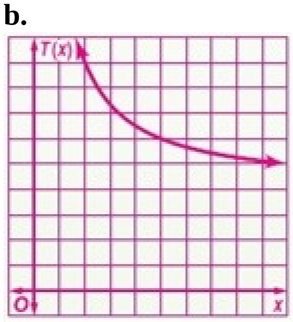
c. Make a table of the function for x = 20, 50, 100, 200, and 400.

d. If it were possible to have an infinite number of trials, what do you think would be the rat's best time? Explain your reasoning.

SOLUTION:

a. Domain: $\{x \mid x \neq 0\}$

b.



c.

x	$T(x)$
20	4.5
50	4.2
100	4.1
200	4.05
400	4.025

d. Sample answer: 4; The fraction approaches 0 as x approaches infinity; 4 + 0 = 4.

63. **CHALLENGE** Simplify $\dfrac{5x^{-2} - \frac{x+1}{x}}{\frac{4}{3-x^{-1}} + 6x^{-1}}$.

SOLUTION:

$$\frac{5x^{-2} - \frac{x+1}{x}}{\frac{4}{3-x^{-1}} + 6x^{-1}} = \frac{\frac{5}{x^2} - \frac{x+1}{x}}{\frac{4}{3-\frac{1}{x}} + \frac{6}{x}}$$

$$= \frac{\frac{5 - x(x+1)}{x^2}}{\frac{4}{\frac{3x-1}{x}} + \frac{6}{x}}$$

$$= \frac{\frac{5 - x^2 - x}{x^2}}{\frac{4x(x) + 6(3x-1)}{x(3x-1)}}$$

$$= \frac{(5 - x^2 - x)(3x-1)}{x(4x^2 + 18x - 6)}$$

$$= \frac{15x - 3x^3 - 3x^2 - 5 + x^2 + x}{4x^3 + 18x^2 - 6x}$$

$$= \frac{-3x^3 - 2x^2 + 16x - 5}{4x^3 + 18x^2 - 6x}$$

64. **CCSS ARGUMENTS** The sum of any two rational numbers is always a rational number. So, the set of rational numbers is said to be closed under addition. Determine whether the set of rational expressions is closed under addition, subtraction, multiplication, and division by a nonzero rational expression. Justify your reasoning.

SOLUTION:

Sample answer: The set of rational expressions is closed under all of these operations because the sum, difference, product, and quotient of two rational expressions is a rational expression.

65. **OPEN ENDED** Write three monomials with an LCM of $180a^4b^6c$.

SOLUTION:

Sample answer: $20a^4b^2c$, $15ab^6$, $9abc$

66. **WRITING IN MATH** Write a how-to manual for adding rational expressions that have unlike denominators.

SOLUTION:

Sample answer: First, factor the denominators of all of the expressions. Find the LCD of the denominators. Convert each expression so they all have the LCD. Add or subtract the numerators. Then simplify. It is the same.

67. **PROBABILITY** A drawing is to be held to select the winner of a new bike. There are 100 seniors, 150 juniors, and 200 sophomores who had correct entries. The drawing will contain 3 tickets for each senior name, 2 for each junior, and 1 for each sophomore. What is the probability that a senior's ticket will be chosen?

A $\frac{1}{8}$

B $\frac{2}{9}$

C $\frac{2}{7}$

D $\frac{3}{8}$

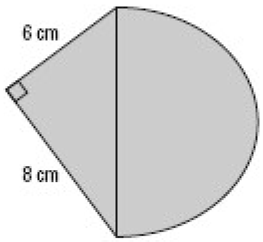
SOLUTION:

Number of senior tickets = 100(3) or 300.
Number of junior tickets = 150(2) or 300.
Number of sophomore tickets = 200(1) or 200.

$$\begin{aligned} P(\text{senior ticket}) &= \frac{300}{300+300+200} \\ &= \frac{300}{800} \\ &= \frac{3}{8} \end{aligned}$$

So, the correct choice is D.

68. **SHORT RESPONSE** Find the area of the figure.



SOLUTION:

$A =$ Area of the triangle + Area of the semi circle

$$= \frac{1}{2}(8)(6) + \frac{1}{2}\pi\left(\frac{\sqrt{8^2+6^2}}{2}\right)^2$$

$$= 24 + \frac{1}{2}\pi(25)$$

$$= 24 + 12.5\pi \text{ cm}^2$$

69. **SAT/ACT** If Mauricio receives *b* books in addition to the number of books he had, he will have *t* times as many as he had originally. In terms of *b* and *t*, how many books did Mauricio have at the beginning?

F $\frac{b}{t-1}$

G $\frac{b}{t+1}$

H $\frac{t+1}{b}$

J $\frac{b}{t}$

K $\frac{t}{b}$

SOLUTION:

Let *x* be the number of books he had originally.

Therefore, $x + b = tx$.

$$tx - x = b$$

$$x = \frac{b}{t-1}$$

The correct choice is F.

70. If $\frac{2a}{a}+\frac{1}{a}=4,$ then a = ___.

A $-\frac{1}{8}$

B $\frac{1}{2}$

C $\frac{1}{8}$

D 2

SOLUTION:

$$\frac{2\cancel{a}}{\cancel{a}}+\frac{1}{a}=4$$
$$2+\frac{1}{a}=4$$
$$\frac{1}{a}=2$$
$$a=\frac{1}{2}$$

The correct choice is B.

Simplify each expression.

71. $\frac{-4ab}{21c}\cdot\frac{14c^2}{22a^2}$

SOLUTION:

$$\frac{-4ab}{\underset{3}{\cancel{21}}\,c}\cdot\frac{\overset{2}{\cancel{14}}\,c^2}{\underset{11}{\cancel{22}}\,a^2}=-\frac{4bc}{33a}$$

72. $\frac{x^2-y^2}{6y}\div\frac{x+y}{36y^2}$

SOLUTION:

Flip the second expression and multiply.

$$\frac{x^2-y^2}{6y}\div\frac{x+y}{36y^2}=\frac{x^2-y^2}{6y}\cdot\frac{36y^2}{x+y}$$
$$=\frac{(x+y)(x-y)}{6y}\cdot\frac{36y^2}{x+y}$$
$$=6y(x-y)$$

73. $\frac{n^2-n-12}{n+2}\div\frac{n-4}{n^2-4n-12}$

SOLUTION:

Flip the second expression and multiply.

$$\frac{n^2-n-12}{n+2}\div\frac{n-4}{n^2-4n-12}$$
$$=\frac{n^2-n-12}{n+2}\cdot\frac{n^2-4n-12}{n-4}$$
$$=\frac{(n-4)(n+3)}{n+2}\cdot\frac{(n-6)(n+2)}{n-4}$$
$$=(n+3)(n-6)$$

74. **BIOLOGY** Bacteria usually reproduce by a process known as *binary fission*. In this type of reproduction, one bacterium divides, forming two bacteria. Under ideal conditions, some bacteria reproduce every 20 minutes.

a. Find the constant k for this type of bacterium under ideal conditions.

b. Write the equation for modeling the exponential growth of this bacterium.

SOLUTION:

a. Substitute 2 for y, 1 for a, 20 for t in the equation $y = ae^{kt}$.

$$\begin{aligned} 2 &= 1\left(e^{k(20)}\right) \\ \ln 2 &= \ln\left(e^{20k}\right) \\ \ln 2 &= 20k \\ k &= \frac{\ln 2}{20} \\ k &\approx 0.0347 \end{aligned}$$

b. Substitute k = 0.0347 in the equation $y = ae^{kt}$.

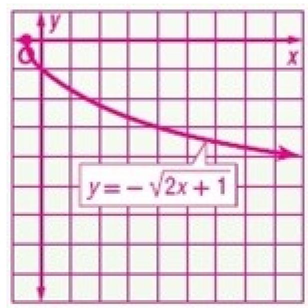
$$y = ae^{0.0347t}$$

Graph each function. State the domain and range of each function.

75. $y = -\sqrt{2x+1}$

SOLUTION:

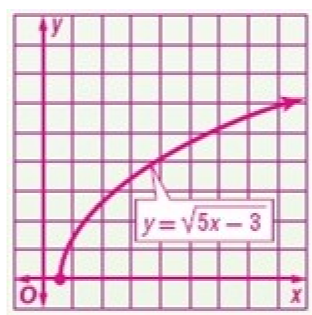
$D = \{x \mid x \geq -0.5\}$, $R = \{y \mid y \leq 0\}$



76. $y = \sqrt{5x-3}$

SOLUTION:

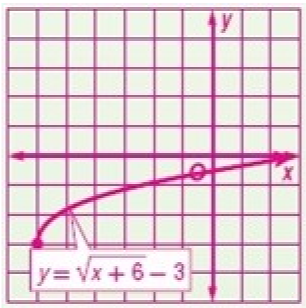
$D = \{x \mid x \geq 0.6\}$, $R = \{y \mid y \geq 0\}$



77. $y = \sqrt{x+6} - 3$

SOLUTION:

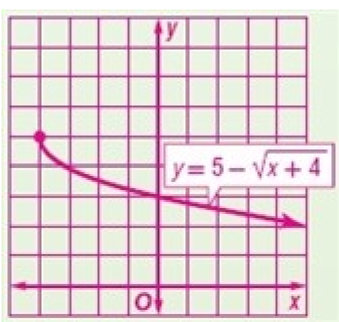
$D = \{x \mid x \geq -6\}$, $R = \{y \mid y \geq -3\}$



78. $y = 5 - \sqrt{x+4}$

SOLUTION:

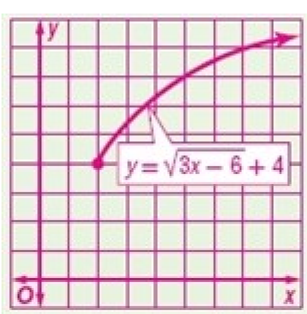
$D = \{x \mid x \geq -4\}$, $R = \{y \mid y \leq 5\}$



79. $y = \sqrt{3x-6} + 4$

SOLUTION:

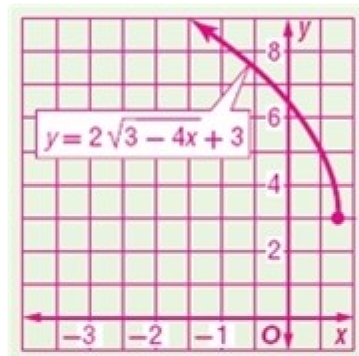
$D = \{x \mid x \geq 2\}$, $R = \{y \mid y \geq 4\}$



80. $y = 2\sqrt{3-4x} + 3$

SOLUTION:

$D = \{x \mid x \leq 0.75\}$, $R = \{y \mid y \geq 3\}$



Solve each equation. State the number and type of roots.

81. $3x + 8 = 0$

SOLUTION:

$$
\begin{aligned}
3x + 8 &= 0 \\
3x &= -8 \\
x &= -\frac{8}{3}
\end{aligned}
$$

There is only one real root.

82. $2x^2 - 5x + 12 = 0$

SOLUTION:

Use the quadratic formula.

$$x = \frac{-5 \pm \sqrt{25 - 4(2)(12)}}{4}$$
$$= \frac{-5 \pm \sqrt{25 - 96}}{4}$$
$$= \frac{-5 \pm i\sqrt{71}}{4}$$

There are two imaginary roots.

83. $x^3 + 9x = 0$

SOLUTION:

$$x(x^2 + 9) = 0$$
$$x = 0 \quad \text{or} \quad x^2 = -9$$
$$x = \pm 3i$$

There is one real root and two imaginary roots.

84. $x^4 - 81 = 0$

SOLUTION:

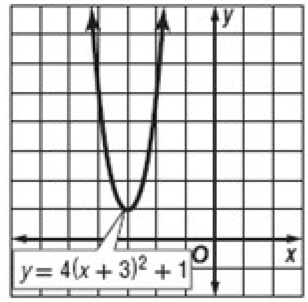
$$(x^2 + 9)(x^2 - 9) = 0$$
$$x^2 + 9 = 0 \quad \text{or} \quad x^2 - 9 = 0$$
$$x = \pm 3i \qquad x = \pm 3$$

There are two real roots and two imaginary roots.

Graph each function.

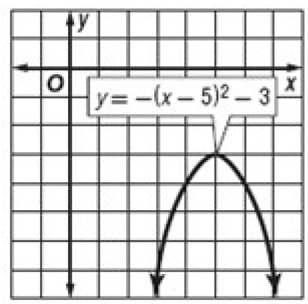
85. $y = 4(x + 3)^2 + 1$

SOLUTION:



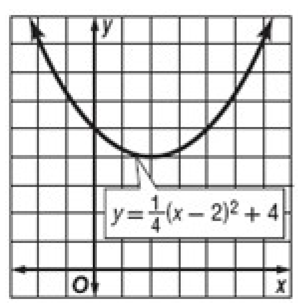
86. $y = -(x - 5)^2 - 3$

SOLUTION:



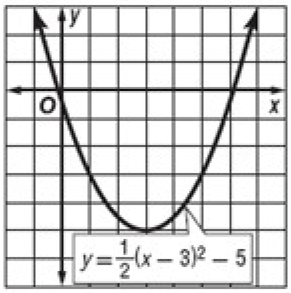
87. $y = \frac{1}{4}(x - 2)^2 + 4$

SOLUTION:



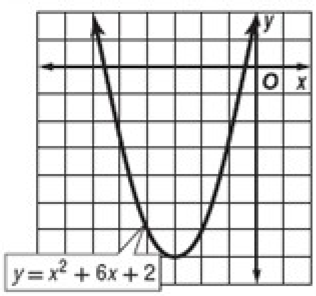
88. $y = \frac{1}{2}(x-3)^2 - 5$

SOLUTION:



89. $y = x^2 + 6x + 2$

SOLUTION:



90. $y = x^2 - 8x + 18$

SOLUTION: