8-4 Graphing Rational Functions

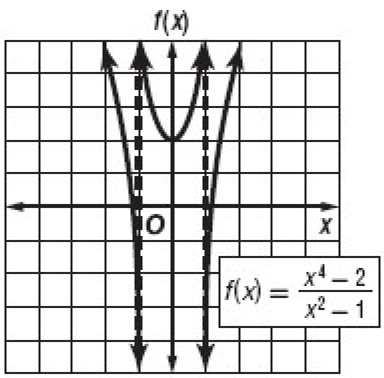
Graph each function.

1. $f(x)=\frac{x^4-2}{x^2-1}$

SOLUTION:

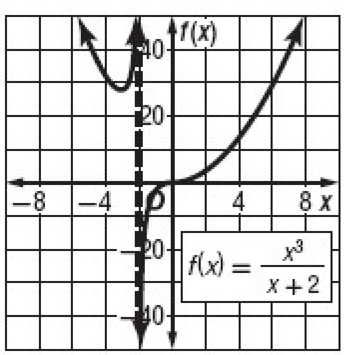
$$x^2-1=0$$
$$(x+1)(x-1)=0$$
$$x=-1 \text{ or } x=1$$

The vertical asymptotes are at $x=-1$ and $x=1$.



2. $f(x)=\frac{x^3}{x+2}$

SOLUTION:
The vertical asymptote is at $x=-2$.



3. **CCSS REASONING** Eduardo is a kicker for his high school football team. So far this season, he has made 7 out of 11 field goals. He would like to improve his field goal percentage. If he can make x consecutive field goals, his field goal percentage can be determined using the function $P(x)=\frac{7+x}{11+x}$.

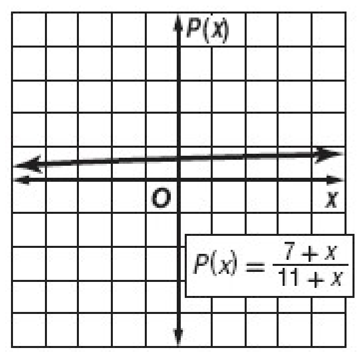
a. Graph the function.

b. What part of the graph is meaningful in the context of this problem?

c. Describe the meaning of the intercept of the vertical axis.

d. What is the equation of the horizontal asymptote? Explain its meaning with respect to Eduardo's field goal percentage.

SOLUTION:
a.



b. the part in the first quadrant

c. It represents his original field goal percentage of 63.6%.

d. $y=1$; this represents 100% which he cannot achieve because he has already missed 4 field goals.

8-4 Graphing Rational Functions

Graph each function.

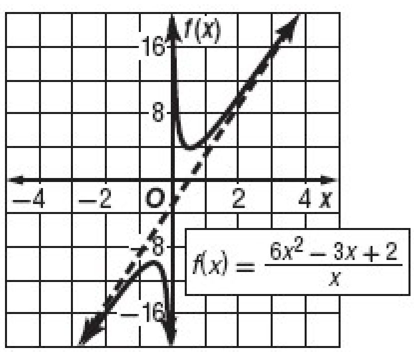
4. $f(x) = \dfrac{6x^2 - 3x + 2}{x}$

SOLUTION:

The vertical asymptote is at $x = 0$.

$$\begin{array}{r|l} & 6x - 3 \\ x & 6x^2 - 3x + 2 \\ & \underline{6x^2} \\ & \quad -3x + 2 \\ & \quad \underline{-3x} \\ & \qquad 2 \end{array}$$

The oblique asymptote is $f(x) = 6x - 3$.



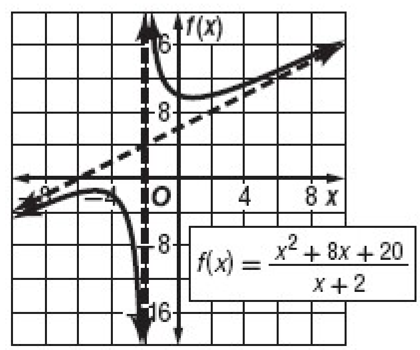
5. $f(x) = \dfrac{x^2 + 8x + 20}{x + 2}$

SOLUTION:

The vertical asymptote is at $x = -2$.

$$\begin{array}{r|l} & x + 6 \\ x + 2 & x^2 + 8x + 20 \\ & \underline{x^2 + 2x} \\ & \quad 6x + 20 \\ & \quad \underline{6x + 12} \\ & \qquad 8 \end{array}$$

The oblique asymptote is $f(x) = x + 6$.

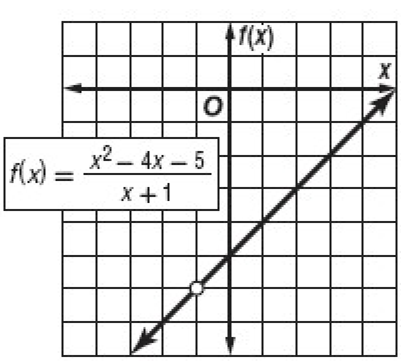


6. $f(x) = \frac{x^2 - 4x - 5}{x + 1}$

SOLUTION:

$$f(x) = \frac{(x+1)(x-5)}{x+1}$$
$$= x - 5 \quad x \neq -1$$

Therefore, the graph of $f(x) = \frac{x^2 - 4x - 5}{x + 1}$ is the graph of $f(x) = x - 5$ with a hole at $x = -1$.

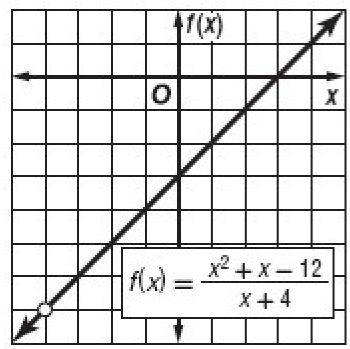


7. $f(x) = \frac{x^2 + x - 12}{x + 4}$

SOLUTION:

$$f(x) = \frac{(x+4)(x-3)}{x+4}$$
$$= x - 3, \ x \neq -4$$

Therefore, the graph of $f(x) = \frac{x^2 + x - 12}{x + 4}$ is the graph of $f(x) = x - 3$ with a hole at $x = -4$.



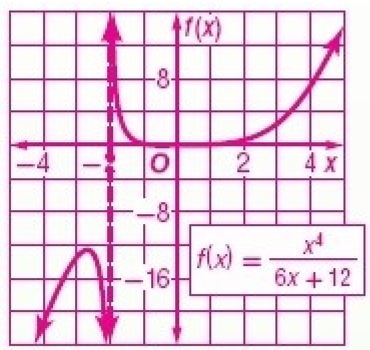
Graph each function.

8. $f(x)=\frac{x^4}{6x+12}$

SOLUTION:

$$\begin{aligned}6x+12&=0\\6x&=-12\\x&=-2\end{aligned}$$

The vertical asymptote is at $x = -2$.

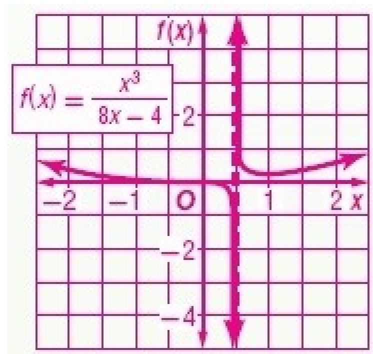


9. $f(x)=\frac{x^3}{8x-4}$

SOLUTION:

$$\begin{aligned}8x-4&=0\\8x&=4\\x&=\frac{4}{8}\\x&=\frac{1}{2}\end{aligned}$$

The vertical asymptote is at $x=\frac{1}{2}$.



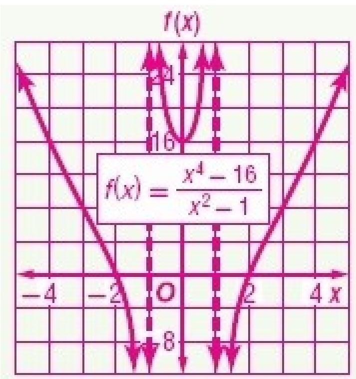
8-4 Graphing Rational Functions

10. $f(x)=\frac{x^4-16}{x^2-1}$

SOLUTION:

$$x^2-1=0$$
$$x^2=1$$
$$x=\pm 1$$

The vertical asymptotes are at $x = 1$ and $x = -1$.

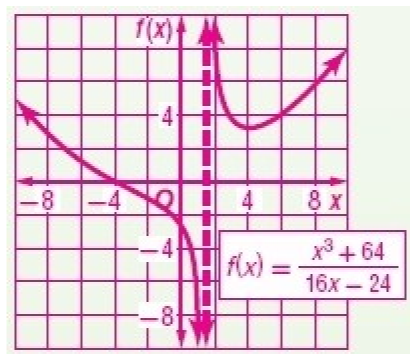


11. $f(x)=\frac{x^3+64}{16x-24}$

SOLUTION:

$$16x-24=0$$
$$16x=24$$
$$x=\frac{24}{16}$$
$$x=\frac{3}{2}$$

The vertical asymptote is at $x=\frac{3}{2}$.



12. **SCHOOL SPIRIT** As president of Student Council, Brandy is getting T-shirts made for a pep rally. Each T-shirt costs $9.50, and there is a set-up fee of $75. The student council plans to sell the shirts, but each of the 15 council members will get one for free.

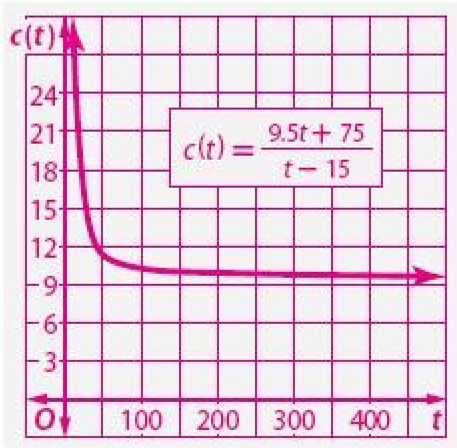
a. Write a function for the average cost of a T-shirt to be sold. Graph the function.

b. What is the average cost if 200 shirts are ordered? if 500 shirts are ordered?

c. How many T-shirts must be ordered to bring the average cost under $9.75?

SOLUTION:

a. $c(t)=\frac{9.5t+75}{t-15}$;



b. The average cost for 200 shirts is:

$$c(200) = \frac{9.5(200) + 75}{200 - 15} \approx \$10.68$$

The average cost for 500 shirts is:

$$c(500) = \frac{9.5(500) + 75}{500 - 15} \approx \$9.95$$

c.

$$\$9.75 > \frac{9.5t + 75}{t - 15}$$
$$9.75t - 146.25 > 9.5t + 75$$
$$t > 885$$

The number of T-shirts ordered to bring the average cost under $9.75 is more than 885.

Graph each function.

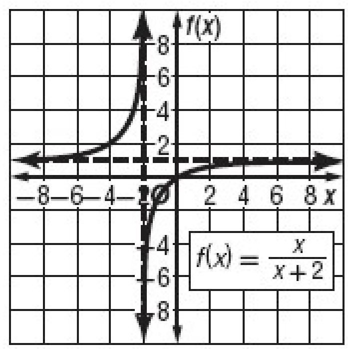
13. $f(x) = \frac{x}{x+2}$

SOLUTION:

$$x + 2 = 0$$
$$x = -2$$

The vertical asymptote is at $x = -2$.

The degrees of the numerator and denominator expressions are same. Therefore, the horizontal asymptote is at $y = 1$.



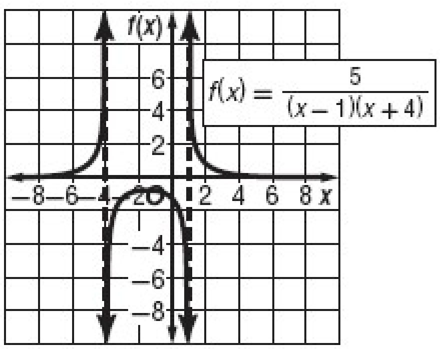
14. $f(x)=\frac{5}{(x-1)(x+4)}$

SOLUTION:

$$(x-1)(x+4)=0$$
$$x-1=0 \text{ or } x+4=0$$
$$x=1 \text{ or } x=-4$$

The vertical asymptotes are at $x = 1$ and $x = -4$.

The horizontal asymptote is at $y = 0$.



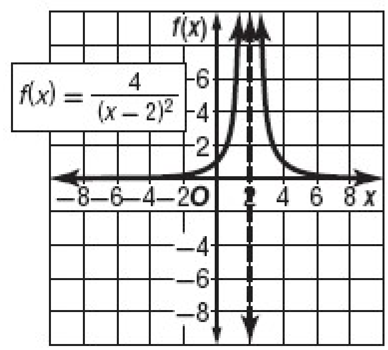
15. $f(x)=\frac{4}{(x-2)^2}$

SOLUTION:

$$(x-2)^2=0$$
$$x-2=0$$
$$x=2$$

The vertical asymptote is at $x = 2$.

The horizontal asymptote is at $y = 0$.



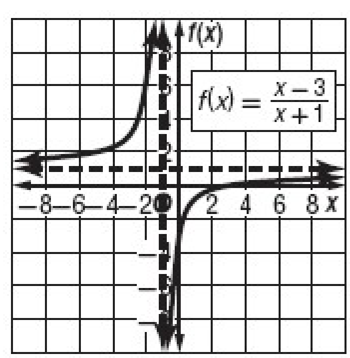
16. $f(x)=\frac{x-3}{x+1}$

SOLUTION:

$$x+1=0$$
$$x=-1$$

The vertical asymptote is at $x = -1$.

The horizontal asymptote is at $y = 1$.



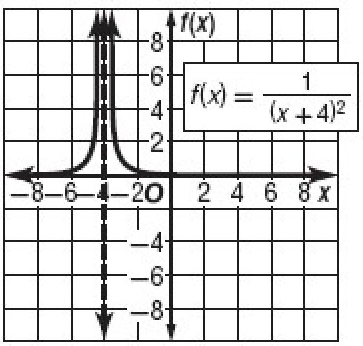
17. $f(x)=\frac{1}{(x+4)^2}$

SOLUTION:

$$\begin{aligned}(x+4)^2&=0\\ x+4&=0\\ x&=-4\end{aligned}$$

The vertical asymptote is at $x = -4$.

The horizontal asymptote is at $y = 0$.

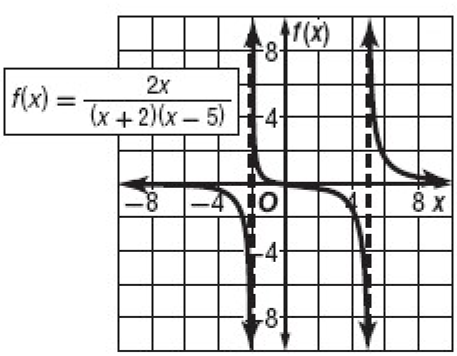


18. $f(x)=\frac{2x}{(x+2)(x-5)}$

SOLUTION:

$$\begin{aligned}(x+2)(x-5)&=0\\ x+2=0 \text{ or } x-5&=0\\ x=-2 \text{ or } x&=5\end{aligned}$$

The vertical asymptotes are at $x = -2$ and $x = 5$.



19. $f(x)=\frac{(x-4)^2}{x+2}$

SOLUTION:

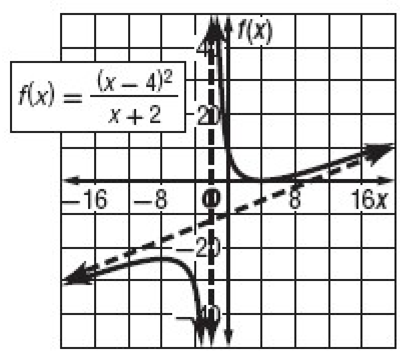
$$\begin{aligned}x+2&=0\\ x&=-2\end{aligned}$$

The vertical asymptote is at $x = -2$.

$$\begin{aligned}f(x)&=\frac{(x-4)^2}{x+2}\\ &=\frac{x^2-8x+16}{x+2}\end{aligned}$$

$$\begin{array}{r} x-10 \\ x+2\,\overline{)\,x^2-8x+16} \\ \underline{x^2+2x} \\ -10x+16 \\ \underline{-10x-20} \\ 36 \end{array}$$

The oblique asymptote is $f(x)=x-10$.



20. $f(x) = \frac{(x+3)^2}{x-5}$

SOLUTION:

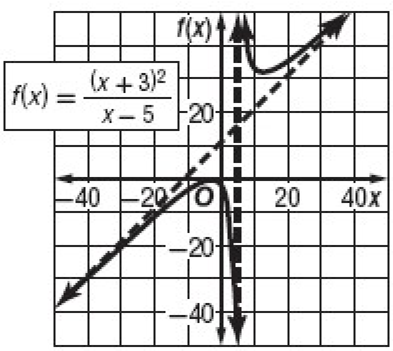
$$x - 5 = 0$$
$$x = 5$$

The vertical asymptote is at $x = 5$.

$$f(x) = \frac{(x+3)^2}{x-5} = \frac{x^2+6x+9}{x-5}$$

$$\begin{array}{r} x+11 \\ x-5\overline{)\,x^2+6x+9} \\ \underline{x^2-5x} \\ 11x+9 \\ \underline{11x-55} \\ 64 \end{array}$$

The oblique asymptote is $f(x) = x + 11$.



21. $f(x) = \frac{x^3+1}{x^2-4}$

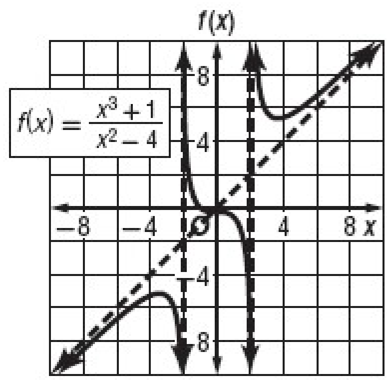
SOLUTION:

$$x^2 - 4 = 0$$
$$x^2 = 4$$
$$x = \pm 2$$

The vertical asymptotes are at $x = 2$ and $x = –2$.

$$\begin{array}{r} x \\ x^2-4\overline{)\,x^3+0x^2+0x+1} \\ \underline{x^3-4x} \\ 4x+1 \end{array}$$

The oblique asymptote is $f(x) = x$.



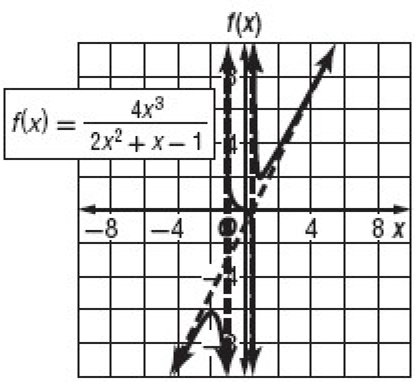
22. $f(x) = \dfrac{4x^3}{2x^2 + x - 1}$

SOLUTION:

$$\begin{aligned} &2x^2 + x - 1 = 0 \\ &2x^2 + 2x - x - 1 = 0 \\ &2x(x+1) - 1(x+1) = 0 \\ &(x+1)(2x-1) = 0 \\ &x + 1 = 0 \text{ or } 2x - 1 = 0 \\ &x = -1 \text{ or } 2x = 1 \\ &\qquad\qquad\quad x = \frac{1}{2} \end{aligned}$$

$$\begin{array}{r|l} & \qquad\qquad 2x - 1 \\ 2x^2 + x - 1 & 4x^3 \\ & \underline{4x^3 + 2x^2 - 2x} \\ & \quad -2x^2 + 2x \\ & \quad \underline{-2x^2 - x + 1} \\ & \qquad\quad 3x - 1 \end{array}$$

The oblique asymptote is $f(x) = 2x - 1$.



23. $f(x) = \dfrac{3x^2 + 8}{2x - 1}$

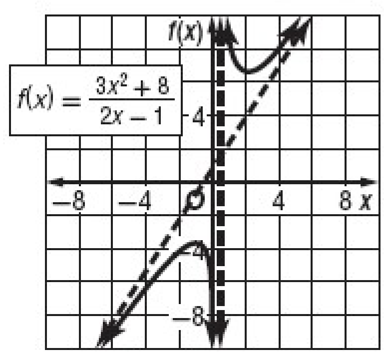
SOLUTION:

$$\begin{aligned} 2x - 1 &= 0 \\ 2x &= 1 \\ x &= \frac{1}{2} \end{aligned}$$

The vertical asymptote is at $x = \frac{1}{2}$.

$$\begin{array}{r|l} & \qquad \frac{3}{2}x + \frac{3}{4} \\ 2x - 1 & 3x^2 + 8 \\ & \underline{3x^2 - \frac{3}{2}x} \\ & \qquad \frac{3}{2}x + 8 \\ & \qquad \underline{\frac{3}{2}x - \frac{3}{4}} \\ & \qquad\quad \left(\frac{35}{4}\right) \end{array}$$

The oblique asymptote is $f(x) = \frac{3}{2}x + \frac{3}{4}$.



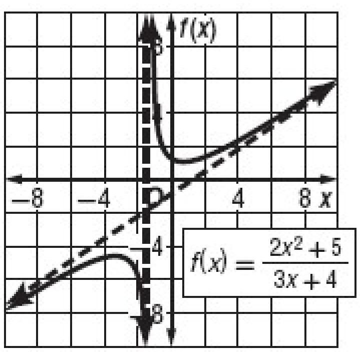
24. $f(x)=\dfrac{2x^2+5}{3x+4}$

SOLUTION:

$$3x+4=0$$
$$3x=-4$$
$$x=-\frac{4}{3}$$

$$\begin{array}{r} \frac{2}{3}x-\frac{8}{9} \\ 3x+4\,\overline{)\,2x^2+5} \\ \underline{2x^2+\frac{8}{3}x} \\ -\frac{8}{3}x+5 \\ \underline{-\frac{8}{3}x-\frac{32}{9}} \end{array}$$

The oblique asymptote is $f(x)=\frac{2}{3}x-\frac{8}{9}$.



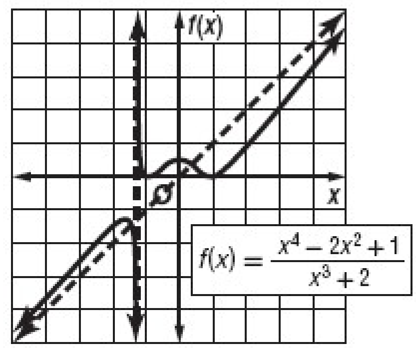
25. $f(x)=\dfrac{x^4-2x^2+1}{x^3+2}$

SOLUTION:

The vertical asymptote is at about $x = -1.26$.

$$\begin{array}{r} x \\ x^3+2\,\overline{)\,x^4-2x^2+1} \\ \underline{x^4+2x} \\ -2x^2-2x \end{array}$$

The oblique asymptote is $f(x)=x$.



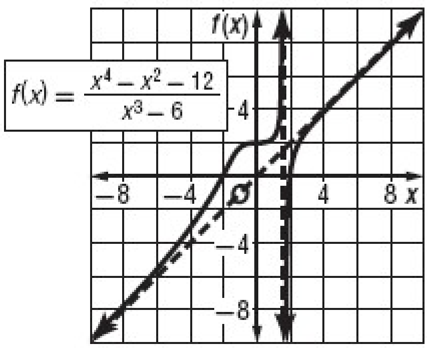
26. $f(x)=\dfrac{x^4-x^2-12}{x^3-6}$

SOLUTION:

The vertical asymptote is at about $x = 1.82$.

$$\begin{array}{r} x \\ x^3-6\overline{\big)\,x^4-x^2-12} \\ \underline{x^4-6x} \\ -x^2+6x \end{array}$$

The oblique asymptote is $f(x)=x$.



27. **CCSS PERSEVERANCE** The current in amperes in an electrical circuit with three resistors in a series is given by the equation $I=\dfrac{V}{R_1+R_2+R_3}$, where V is the voltage in volts in a the circuit and R_1, R_2, and R_3 are the resistances in ohms of the three resistors.

a. Let R_1 be the independent variable, and let I be the dependent variable. Graph the function if $V = 120$ volts, $R_2 = 25$ ohms, and $R_3 = 75$ ohms.

b. Give the equation of the vertical asymptote and the R_1- and I-intercepts of the graph.

c. Find the value of I when the value of R_1 is 140 ohms.

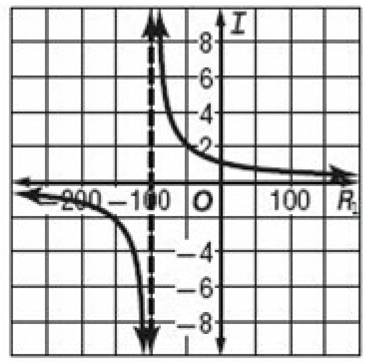
d. What domain and range values are meaningful in the context of the problem?

SOLUTION:

a.

Graph the function

$I(R_1)=\dfrac{120}{R_1+25+75}$ or $I(R_1)=\dfrac{120}{R_1+100}$.



b. The vertical Asymptote is:

$$R_1+100=0$$
$$R_1=-100$$

R_1- Intercept: No

I-intercept: 1.2

c. Substitute 140 for R_1.

$$I(140)=\frac{120}{140+100}$$
$$=0.5 \text{ amperes}$$

d.

Domain: $R_1 \ge 0$;

Range: $0 < I \le 1.2$;

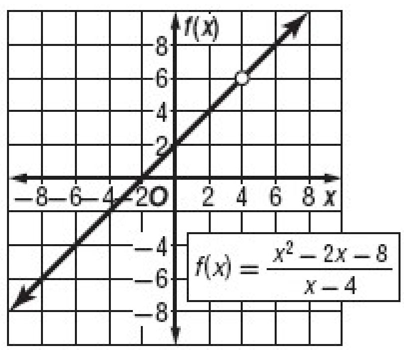
Graph each function.

28. $f(x)=\frac{x^2-2x-8}{x-4}$

SOLUTION:

$$f(x)=\frac{(x+2)(x-4)}{x-4}$$
$$=x+2,\quad x\neq 4$$

Therefore, the graph of $f(x)=\frac{x^2-2x-8}{x-4}$ is the graph of $f(x)=x+2$ with a hole at $x = 4$.

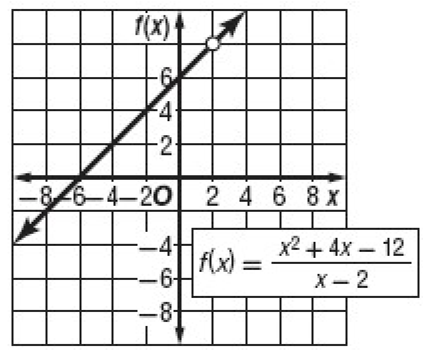


29. $f(x)=\frac{x^2+4x-12}{x-2}$

SOLUTION:

$$f(x)=\frac{(x+6)(x-2)}{x-2}$$
$$=x+6,\quad x\neq 2$$

Therefore, the graph of $f(x)=\frac{x^2+4x-12}{x-2}$ is the graph of $f(x)=x+6$ with a hole at $x = 2$.

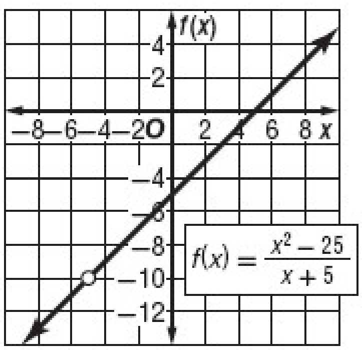


30. $f(x) = \dfrac{x^2 - 25}{x + 5}$

SOLUTION:

$$f(x) = \frac{(x+5)(x-5)}{x+5}$$
$$= x - 5, \quad x = -5$$

Therefore, the graph of $f(x) = \dfrac{x^2 - 25}{x + 5}$ is the graph of $f(x) = x - 5$ with a hole at $x = -5$.

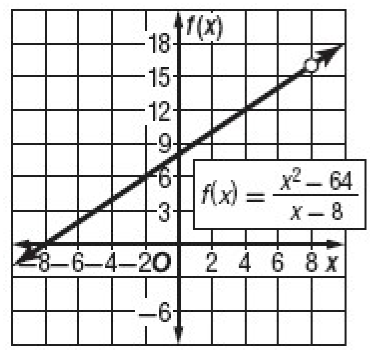


31. $f(x) = \dfrac{x^2 - 64}{x - 8}$

SOLUTION:

$$f(x) = \frac{(x-8)(x+8)}{x-8}$$
$$= x + 8, \quad x \neq 8$$

Therefore, the graph of $f(x) = \dfrac{x^2 - 64}{x - 8}$ is the graph of $f(x) = x + 8$ with a hole at $x = 8$.

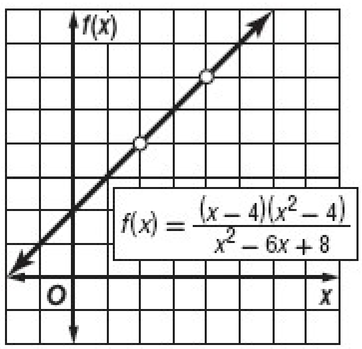


32. $f(x)=\dfrac{(x-4)(x^2-4)}{x^2-6x+8}$

SOLUTION:

$$f(x)=\frac{(x-4)(x-2)(x+2)}{(x-4)(x-2)}$$
$$=x+2,\quad x\neq 4,\,2$$

Therefore, the graph of $f(x)=\dfrac{(x-4)(x^2-4)}{x^2-6x+8}$ is the graph of $f(x)=x+2$ with the holes at x = 4 and 2.

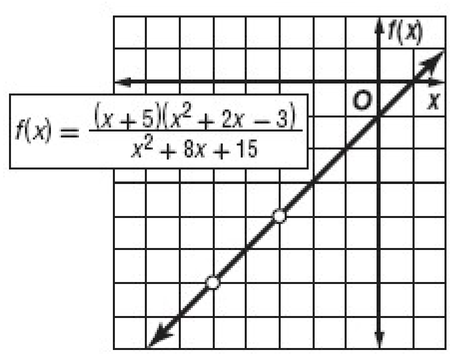


33. $f(x)=\dfrac{(x+5)(x^2+2x-3)}{x^2+8x+15}$

SOLUTION:

$$f(x)=\frac{(x+5)(x+3)(x-1)}{(x+3)(x+5)}$$
$$=x-1,\quad x=-3,\,-5$$

Therefore, the graph of $f(x)=\dfrac{(x+5)(x^2+2x-3)}{x^2+8x+15}$ is the graph of $f(x)=x-1$ with the holes at x = –3 and –5.

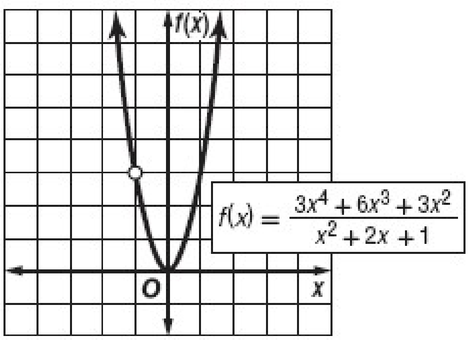


34. $f(x)=\frac{3x^4+6x^3+3x^2}{x^2+2x+1}$

SOLUTION:

$$\begin{aligned} f(x) &= \frac{3x^2\left(x^2+2x+1\right)}{x^2+2x+1} \\ &= 3x^2,\ \ x^2+2x+1\neq 0 \\ &= 3x^2,\ \ x\neq -1 \end{aligned}$$

Therefore, the graph of $f(x)=\frac{3x^4+6x^3+3x^2}{x^2+2x+1}$ is the graph of $f(x)=3x^2$ with a hole at $x = –1$.

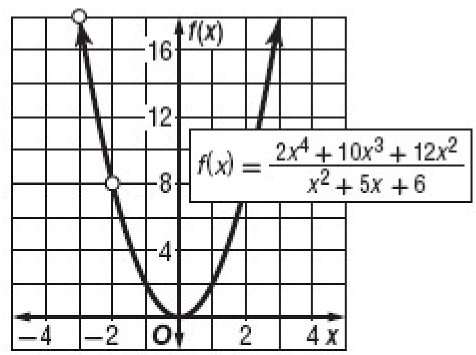


35. $f(x)=\frac{2x^4+10x^3+12x^2}{x^2+5x+6}$

SOLUTION:

$$\begin{aligned} f(x) &= \frac{2x^2(x^2+5x+6)}{x^2+5x+6} \\ &= 2x^2,\ \ x^2+5x+6\neq 0 \\ &= 2x^2,\ \ x\neq -3,\ -2 \end{aligned}$$

Therefore, the graph of $f(x)=\frac{2x^4+10x^3+12x^2}{x^2+5x+6}$ is the graph of $f(x)=2x^2$ with the holes at $x = –3$ and $x = –1$.



36. **BUSINESS** Liam purchased a snow plow for $4500 and plows the parking lots of local businesses. Each time he plows a parking lot, he incurs a cost of $50 for gas and maintenance.

a. Write and graph the rational function representing his average cost per customer as a function of the number of parking lots.

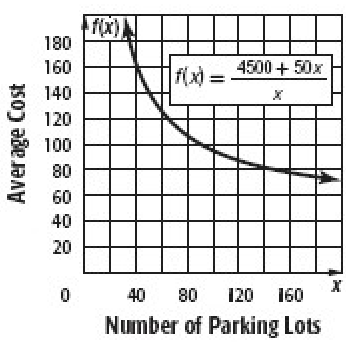
b. What are the asymptotes of the graph?

c. Why is the first quadrant in the graph the only relevant quadrant?

d. How many total parking lots does Liam need to plow for his average cost per parking lot to be less than $80?

SOLUTION:

a. Let x be the number of parking lots. Therefore, the function that represents the average cost per customer is $f(x) = \frac{4500 + 50x}{x}$.



b. The vertical asymptote is $x = 0$ and the horizontal asymptote is $f(x) = 50$.

c. Sample answer: The number of parking lots and the average cost cannot be negative.

d. Substitute 80 for $f(x)$.

$$80 = \frac{4500 + 50x}{x}$$
$$30x = 4500$$
$$x = 150$$

37. **FINANCIAL LITERACY** Kristina bought a new cell phone with Internet access. The phone cost $150, and her monthly usage charge is $30 plus $10 for the Internet access.

a. Write and graph the rational function representing her average monthly cost as a function of the number of months Kristina uses the phone.

b. What are the asymptotes of the graph?

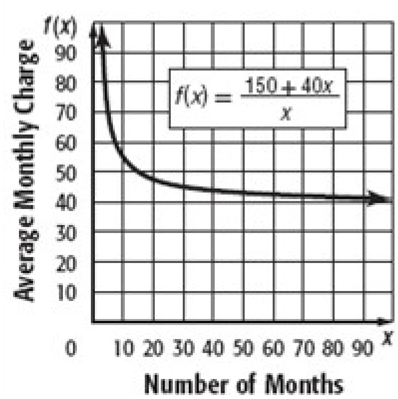
c. Why is the first quadrant in the graph the only relevant quadrant?

d. After how many months will the average monthly charge be $45?

SOLUTION:

a. Let x be the number of months Kristina uses the phone.

Therefore, the function that represents the average monthly costs is $f(x) = \frac{150 + 40x}{x}$.



b. Vertical asymptote: $x = 0$

Horizontal asymptote: $f(x) = 40$

c. Sample answer: The number of months and the average cost cannot have negative values.

d. Substitute $f(x) = 45$.

$$45 = \frac{150 + 40x}{x}$$
$$5x = 150$$
$$x = 30$$

38. **CCSS SENSE-MAKING** Alana plays softball for Centerville High School. So far this season she has gotten a hit 4 out of 12 times at bat. She is determined to improve her batting average. If she can get *x* consecutive hits, her batting average can be determined using $B(x) = \frac{4 + x}{12 + x}$.

a. Graph the function.

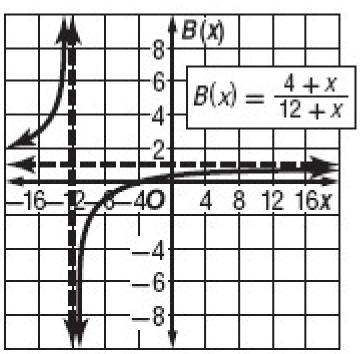
b. What part of the graph is meaningful in the context of the problem?

c. Describe the meaning of the intercept of the vertical axis.

d. What is the equation of the horizontal asymptote? Explain its meaning with respect to Alana's batting average.

SOLUTION:

a.



b. the part in the first quadrant

c. It represents her original batting average of .333.

d. $y = 1$; This represents 100%, which she can never achieve because she has already missed getting a hit 8 times.

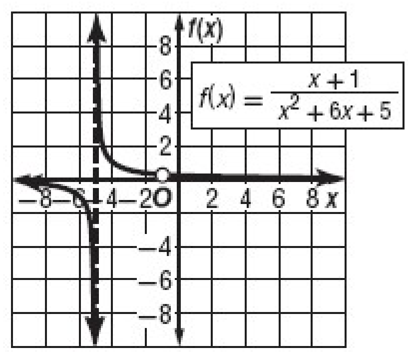
Graph each function.

39. $f(x) = \frac{x + 1}{x^2 + 6x + 5}$

SOLUTION:

$$f(x) = \frac{x + 1}{(x + 5)(x + 1)}$$
$$= \frac{1}{x + 5}, \quad x \neq -1$$

Therefore, the graph of $f(x) = \frac{x + 1}{x^2 + 6x + 5}$ is the graph of $f(x) = \frac{1}{x + 5}$ with the holes at $x = -1$.

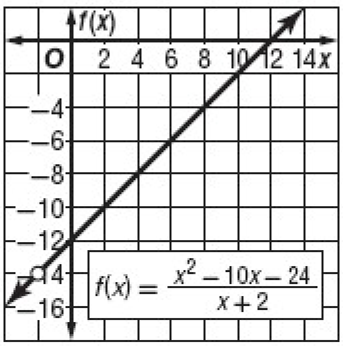


40. $f(x)=\dfrac{x^2-10x-24}{x+2}$

SOLUTION:

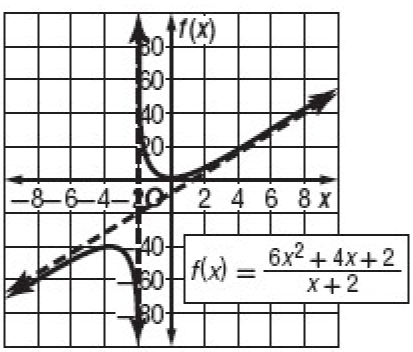
$$f(x)=\frac{(x+2)(x-12)}{x+2}$$
$$=x-12,\quad x\neq-2$$

Therefore, the graph of $f(x)=\dfrac{x^2-10x-24}{x+2}$ is the graph of $f(x)=x-12$ with the holes at $x = –2$.



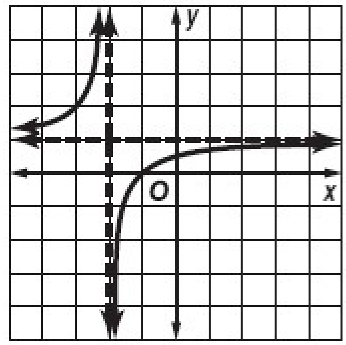
41. $f(x)=\dfrac{6x^2+4x+2}{x+2}$

SOLUTION:
Graph the function.

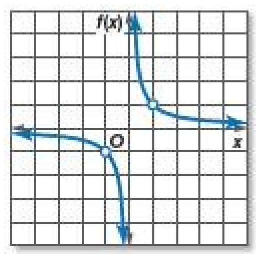


42. **OPEN ENDED** Sketch the graph of a rational function with a horizontal asymptote $y = 1$ and a vertical asymptote $x = –2$.

SOLUTION:
Sample graph:



43. **CHALLENGE** Compare and contrast $g(x)=\dfrac{x^2-1}{x(x^2-2)}$ and $f(x)$ shown.



SOLUTION:
Similarities: Both have vertical asymptotes at $x = 0$. Both approach 0 as x approaches $-\infty$ and approach 0 as x approaches ∞. Differences: $f(x)$ has holes at $x = 1$ and $x = –1$, while $g(x)$ has vertical asymptotes at $x=\sqrt{2}$ and $x=-\sqrt{2}$. $f(x)$ has no zeros, but $g(x)$ has zeros at $x = 1$ and $x = –1$.

44. **REASONING** What is the difference between the graphs of $f(x) = x – 2$ and $g(x)=\dfrac{(x+3)(x-2)}{x+3}$?

SOLUTION:
The graph of $g(x)$ has a hole in it at –3**.**

45. **PROOF** A rational function has an equation of the form $f(x)=\frac{a(x)}{b(x)}$, where $a(x)$ and $b(x)$ are polynomial functions and $b(x) \neq 0$. Show that $f(x)=\frac{x}{a-b}+c$ is a rational function.

SOLUTION:

$$f(x)=\frac{x}{a-b}+c$$

Find the common denominator.

$$\begin{aligned} f(x) &= \frac{x}{a-b}+\frac{c(a-b)}{(a-b)} \\ &= \frac{x+ca-cb}{a-b} \end{aligned}$$

Since $a(x)$ and $b(x)$ are polynomial functions, $f(x)$ is a rational function.

46. **WRITING IN MATH** How can factoring be used to determine the vertical asymptotes or point discontinuity of a rational function?

SOLUTION:

Sample answer: By factoring the denominator of a rational function and determining the values that cause each factor to equal zero you can determine the asymptotes of a rational function. After factoring the numerator and denominator of a rational function, if there is a common factor $x - c$, then there is point discontinuity at $x = c$.

47. **PROBABILITY** Of the 6 courses offered by the music department at her school, Kaila must choose exactly 2 of them. How many different combinations of 2 courses are possible for Kaila if there are no restrictions on which 2 courses she can choose?

A 48

B 18

C 15

D 12

SOLUTION:

$$\begin{aligned} {}_6n_2 &= \frac{6\cdot 5}{2\cdot 1} \\ &= 15 \end{aligned}$$

The correct choice is C.

48. The projected sales of a game cartridge is given by the function $S(p) = \frac{3000}{2p + a}$, where $S(p)$ is the number of cartridges sold, in thousands, p is the price per cartridge, in dollars, and a is a constant.

If 100,000 cartridges are sold at $10 per cartridge, how many cartridges will be sold at $20 per cartridge?

F 20,000

G 50,000

H 60,000

J 150,000

SOLUTION:
Find the value of the constant a.

$$\begin{aligned} 100 &= \frac{3000}{2(10) + a} \\ 20 + a &= 30 \\ a &= 10 \end{aligned}$$

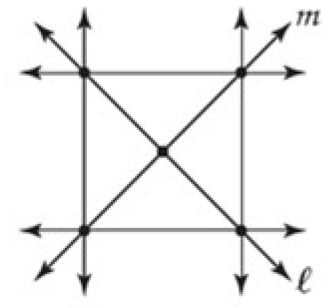
Now,

$$\begin{aligned} S(p) &= \frac{3000}{2(20) + 10} \\ &= \frac{3000}{50} \\ &= 60 \end{aligned}$$

The correct choice is H.

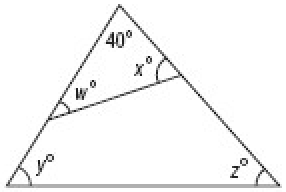
49. **GRIDDED RESPONSE** Five distinct points lie in a plane such that 3 of the points are on line ℓ and 3 of the points are on a different line *m*. What is the total number of lines that can be drawn so that each line passes through exactly 2 of these 5 points?

SOLUTION:



From the figure, 4 lines that can be drawn so that each line passes through exactly 2 points.

50. **GEOMETRY** In the figure below, what is the value of $w + x + y + z$?



A 140

B 280

C 320

D 360

SOLUTION:
By the Triangle Angle-Sum Theorem, $y + z + 40 = 180$ and $w + x + 40 = 180$.

Therefore,

$$y + z = 140;$$
$$w + x = 140;$$

Now,

$$\begin{aligned} w + x + y + z &= 140 + 140 \\ &= 280 \end{aligned}$$

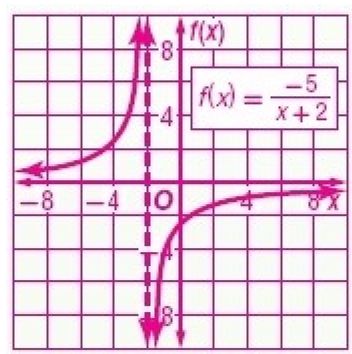
The correct choice B.

Graph each function. State the domain and range.

51. $f(x) = \frac{-5}{x+2}$

SOLUTION:
The vertical asymptote is at $x = -2$.

The horizontal asymptote is at $y = 0$.

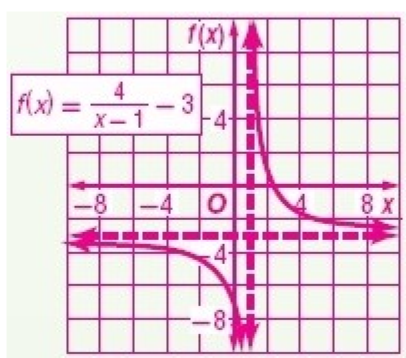


D = $\{x \mid x \neq -2\}$, R = $\{f(x) \mid f(x) \neq 0\}$

52. $f(x) = \frac{4}{x-1} - 3$

SOLUTION:
The vertical asymptote is at $x = 1$.

The horizontal asymptote is at $y = -3$.



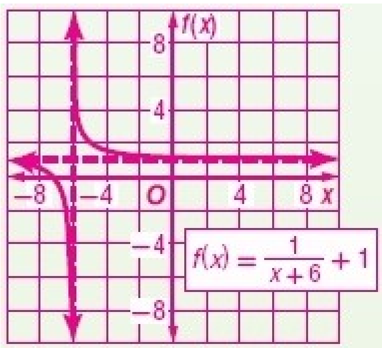
D = $\{x \mid x \neq 1\}$, R = $\{f(x) \mid f(x) \neq -3\}$

53. $f(x)=\frac{1}{x+6}+1$

SOLUTION:

The vertical asymptote is at $x = -6$.

The horizontal asymptote is at $y = 1$.



D = {x | $x \neq -6$}, R = {$f(x)$ | $f(x) \neq 1$}

Simplify each expression.

54. $\frac{m}{m^2-4}+\frac{2}{3m+6}$

SOLUTION:

$$\frac{m}{m^2-4}+\frac{2}{3m+6}=\frac{m}{(m-2)(m+2)}+\frac{2}{3(m+2)}$$

The LCD is $3(m-2)(m+2)$.

$$\frac{m}{(m-2)(m+2)}+\frac{2}{3(m+2)}=\frac{3m+2(m-2)}{3(m-2)(m+2)}$$
$$=\frac{5m-4}{3(m-2)(m+2)}$$

55. $\frac{y}{y+3}-\frac{6y}{y^2-9}$

SOLUTION:

$$\frac{y}{y+3}-\frac{6y}{y^2-9}=\frac{y}{y+3}-\frac{6y}{(y+3)(y-3)}$$

The LCD is $(y+3)(y-3)$.

$$\frac{y}{y+3}-\frac{6y}{(y+3)(y-3)}=\frac{y(y-3)-6y}{(y+3)(y-3)}$$
$$=\frac{y(y-9)}{(y+3)(y-3)}$$

56. $\frac{5}{x^2-3x-28}+\frac{7}{2x-14}$

SOLUTION:

$$\frac{5}{x^2-3x-28}+\frac{7}{2x-14}=\frac{5}{(x-7)(x+4)}+\frac{7}{2(x-7)}$$

The LCD is $2(x-7)(x+4)$.

$$\frac{5}{(x-7)(x+4)}+\frac{7}{2(x-7)}=\frac{2(5)+7(x+4)}{2(x-7)(x+4)}$$
$$=\frac{7x+38}{2(x-7)(x+4)}$$

57. $\frac{d-4}{d^2+2d-8}-\frac{d+2}{d^2-16}$

SOLUTION:

$$\frac{d-4}{d^2+2d-8}-\frac{d+2}{d^2-16}$$
$$=\frac{d-4}{(d+4)(d-2)}-\frac{d+2}{(d-4)(d+4)}$$

The LCD is $(d-4)(d+4)(d+2)$.

$$\frac{d-4}{(d+4)(d-2)}-\frac{d+2}{(d-4)(d+4)}$$
$$=\frac{(d-4)(d-4)-(d+2)(d-2)}{(d-4)(d+4)(d-2)}$$
$$=\frac{d^2-8d+16-d^2+4}{(d-4)(d+4)(d-2)}$$
$$=\frac{-8d+20}{(d-4)(d+4)(d-2)}$$

Simplify each expression.

58. $y^{\frac{5}{3}}\cdot y^{\frac{7}{3}}$

SOLUTION:

$$y^{\frac{5}{3}}\cdot y^{\frac{7}{3}}=y^{\frac{5}{3}+\frac{7}{3}}$$
$$=y^{\frac{12}{3}}$$
$$=y^4$$

59. $x^{\frac{3}{4}}\cdot x^{\frac{9}{4}}$

SOLUTION:

$$x^{\frac{3}{4}}\cdot x^{\frac{9}{4}}=x^{\frac{3}{4}+\frac{9}{4}}$$
$$=x^{\frac{12}{4}}$$
$$=x^3$$

60. $\left(b^{\frac{1}{3}}\right)^{\frac{3}{5}}$

SOLUTION:

$$\left(b^{\frac{1}{3}}\right)^{\frac{3}{5}}=b^{\frac{1}{3}\cdot\frac{3}{5}}$$
$$=b^{\frac{1}{5}}$$

61. $\left(a^{-\frac{2}{3}}\right)^{-\frac{1}{6}}$

SOLUTION:

$$\left(a^{-\frac{2}{3}}\right)^{-\frac{1}{6}}=a^{-\frac{2}{3}\cdot\frac{-1}{6}}$$
$$=a^{\frac{1}{9}}$$

62. **TRAVEL** Mr. and Mrs. Wells are taking their daughter to college. The table shows their distances from home after various amounts of time.

a. Find the average rate of change in their distances from home between 1 and 3 hours after leaving home.

b. Find the average rate of change in their distances from home between 0 and 5 hours after leaving home.

Time (h)	Distance (mi)
0	0
1	55
2	110
3	165
4	165
5	225

SOLUTION:

a. Average rate of change between 1 and 3 hours:

$$\frac{165-55}{3-1}=\frac{110}{2}=55\text{mph}$$

b. Average rate of change between 0 and 5 hours:

$$\frac{225-0}{5-0}=\frac{225}{5}=45\text{mph}$$