8-6 Solving Rational Equations and Inequalities

Solve each equation. Check your solution.

1. $\dfrac{4}{7} + \dfrac{3}{x-3} = \dfrac{53}{56}$

SOLUTION:

$$\frac{4}{7} + \frac{3}{x-3} = \frac{53}{56}$$

$$56(x-3)\left(\frac{4}{7}\right) + 56(x-3)\frac{3}{(x-3)} = 56(x-3)\frac{53}{56}$$

$$32(x-3) + 56(3) = 53(x-3)$$

$$32x - 96 + 168 = 53x - 159$$

$$32x - 53x = -159 + 96 - 168$$

$$-21x = -231$$

$$x = 11$$

Check:

$$\frac{4}{7} + \frac{3}{11-3} \overset{?}{=} \frac{53}{56}$$

$$\frac{4}{7} + \frac{3}{8} \overset{?}{=} \frac{53}{56}$$

$$\frac{4(8) + 3(7)}{56} \overset{?}{=} \frac{53}{56}$$

$$\frac{32 + 21}{56} \overset{?}{=} \frac{53}{56}$$

$$\frac{53}{56} = \frac{53}{56} \checkmark$$

The solution is 11.

2. $\dfrac{7}{3} - \dfrac{3}{x-5} = \dfrac{19}{12}$

SOLUTION:

$$\frac{7}{3} - \frac{3}{x-5} = \frac{19}{12}$$

$$12(x-5)\frac{7}{3} - 12(x-5)\frac{3}{(x-5)} = 12(x-5)\frac{19}{12}$$

$$28(x-5) - 36 = 19x - 95$$

$$28x - 140 - 36 = 19x - 95$$

$$28x - 19x = -95 + 140 + 36$$

$$9x = 81$$

$$x = 9$$

Check:

$$\frac{7}{3} - \frac{3}{9-5} \overset{?}{=} \frac{19}{12}$$

$$\frac{7}{3} - \frac{3}{4} \overset{?}{=} \frac{19}{12}$$

$$\frac{7(4) - 3(3)}{12} \overset{?}{=} \frac{19}{12}$$

$$\frac{28 - 9}{12} \overset{?}{=} \frac{19}{12}$$

$$\frac{19}{12} = \frac{19}{12} \checkmark$$

The solution is 9.

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3. $\dfrac{10}{2x+1} + \dfrac{4}{3} = 2$

SOLUTION:

$$\dfrac{10}{2x+1} + \dfrac{4}{3} = 2$$

$$\dfrac{10(3) + 4(2x+1)}{3(2x+1)} = 2$$

$$\dfrac{30 + 8x + 4}{3(2x+1)} = 2$$

$$8x + 34 = 6(2x+1)$$

$$8x + 34 = 12x + 6$$

$$-4x = 6 - 34$$

$$-4x = -28$$

$$x = 7$$

Check:

$$\dfrac{10}{2(7)+1} + \dfrac{4}{3} \overset{?}{=} 2$$

$$\dfrac{10}{14+1} + \dfrac{4}{3} \overset{?}{=} 2$$

$$\dfrac{10}{15} + \dfrac{4}{3} \overset{?}{=} 2$$

$$\dfrac{2}{3} + \dfrac{4}{3} \overset{?}{=} 2$$

$$\dfrac{2+4}{3} \overset{?}{=} 2$$

$$\dfrac{6}{3} \overset{?}{=} 2$$

$$3 = 3 \checkmark$$

The solution is 7.

4. $\dfrac{11}{4} - \dfrac{5}{y+3} = \dfrac{23}{12}$

SOLUTION:

$$\dfrac{11}{4} - \dfrac{5}{y+3} = \dfrac{23}{12}$$

$$12(y+3)\dfrac{11}{4} - 12(y+3)\dfrac{5}{(y+3)} = 12(y+3)\dfrac{23}{12}$$

$$33(y+3) - 60 = 23(y+3)$$

$$33y + 99 - 60 = 23y + 69$$

$$33y - 23y = 60 + 69 - 99$$

$$10y = 30$$

$$y = 3$$

Check:

$$\dfrac{11}{4} - \dfrac{5}{3+3} \overset{?}{=} \dfrac{23}{12}$$

$$\dfrac{11}{4} - \dfrac{5}{6} \overset{?}{=} \dfrac{23}{12}$$

$$\dfrac{11(6) - 5(4)}{24} \overset{?}{=} \dfrac{23}{12}$$

$$\dfrac{66 - 20}{24} \overset{?}{=} \dfrac{23}{12}$$

$$\dfrac{46}{24} \overset{?}{=} \dfrac{23}{12}$$

$$\dfrac{23}{12} = \dfrac{23}{12} \checkmark$$

The solution is 3.

5. $\dfrac{8}{x-5} - \dfrac{9}{x-4} = \dfrac{5}{x^2 - 9x + 20}$

6. $\dfrac{14}{x+3} + \dfrac{10}{x-2} = \dfrac{122}{x^2 + x - 6}$

SOLUTION:

$$\dfrac{8}{x-5} - \dfrac{9}{x-4} = \dfrac{5}{x^2 - 9x + 20}$$

$$\dfrac{8(x-4) - 9(x-5)}{(x-5)(x-4)} = \dfrac{5}{x^2 - 9x + 20}$$

$$\dfrac{8x - 32 - 9x + 45}{x^2 - 9x + 20} = \dfrac{5}{x^2 - 9x + 20}$$

$$-x + 13 = 5$$

$$-x = 5 - 13$$

$$-x = -8$$

$$x = 8$$

Check:

$$\dfrac{8}{8-5} - \dfrac{9}{8-4} \overset{?}{=} \dfrac{5}{8^2 - 72 + 20}$$

$$\dfrac{8}{3} - \dfrac{9}{4} \overset{?}{=} \dfrac{5}{64 - 72 + 20}$$

$$\dfrac{8(4) - 9(3)}{12} \overset{?}{=} \dfrac{5}{12}$$

$$\dfrac{32 - 27}{12} \overset{?}{=} \dfrac{5}{12}$$

$$\dfrac{5}{12} = \dfrac{5}{12} \checkmark$$

The solution is 8.

SOLUTION:

$$\dfrac{14}{x+3} + \dfrac{10}{x-2} = \dfrac{122}{x^2 + x - 6}$$

$$\dfrac{14(x-2) + 10(x+3)}{(x+3)(x-2)} = \dfrac{122}{x^2 + x - 6}$$

$$\dfrac{14x - 28 + 10x + 30}{x^2 + x - 6} = \dfrac{122}{x^2 + x - 6}$$

$$\dfrac{24x + 2}{x^2 + x - 6} = \dfrac{122}{x^2 + x - 6}$$

$$24x + 2 = 122$$

$$24x = 122 - 2$$

$$24x = 120$$

$$x = \dfrac{120}{24}$$

$$x = 5$$

Check:

$$\dfrac{14}{5+3} + \dfrac{10}{5-2} \overset{?}{=} \dfrac{122}{5^2 + 5 - 6}$$

$$\dfrac{14}{8} + \dfrac{10}{3} \overset{?}{=} \dfrac{122}{24}$$

$$\dfrac{14(3) + 10(8)}{24} \overset{?}{=} \dfrac{122}{24}$$

$$\dfrac{42 + 80}{24} \overset{?}{=} \dfrac{122}{24}$$

$$\dfrac{122}{24} = \dfrac{122}{24} \checkmark$$

The solution is $x = 5$.

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7. $\dfrac{14}{x-8} - \dfrac{5}{x-6} = \dfrac{82}{x^2-14x+48}$

SOLUTION:

$$\frac{14}{x-8} - \frac{5}{x-6} = \frac{82}{x^2-14x+48}$$

$$\frac{14(x-6)-5(x-8)}{(x-8)(x-6)} = \frac{82}{x^2-14x+48}$$

$$\frac{14x-84-5x+40}{x^2-14x+48} = \frac{82}{x^2-14x+48}$$

$$9x-44 = 82$$

$$9x = 126$$

$$x = \frac{126}{9}$$

$$x = 14$$

Check:

$$\frac{14}{14-8} - \frac{5}{14-6} \overset{?}{=} \frac{82}{14^2-14(14)+48}$$

$$\frac{14}{6} - \frac{5}{8} \overset{?}{=} \frac{82}{196-196+48}$$

$$\frac{14(8)-5(6)}{48} \overset{?}{=} \frac{82}{48}$$

$$\frac{112-30}{48} \overset{?}{=} \frac{82}{48}$$

$$\frac{82}{48} = \frac{82}{48} \checkmark$$

The solution is 14.

8. $\dfrac{5}{x+2} - \dfrac{3}{x-2} = \dfrac{12}{x^2-4}$

SOLUTION:

$$\frac{5}{x+2} - \frac{3}{x-2} = \frac{12}{x^2-4}$$

$$\frac{5(x-2)-3(x+2)}{(x+2)(x-2)} = \frac{12}{x^2-4}$$

$$\frac{5x-10-3x-6}{x^2-4} = \frac{12}{x^2-4}$$

$$\frac{2x-16}{x^2-4} = \frac{12}{x^2-4}$$

$$2x-16 = 12$$

$$2x = 12+16$$

$$2x = 28$$

$$x = 14$$

Check:

$$\frac{5}{14+2} - \frac{3}{14-2} \overset{?}{=} \frac{12}{14^2-4}$$

$$\frac{5}{16} - \frac{3}{12} \overset{?}{=} \frac{12}{196-4}$$

$$\frac{5(12)-3(16)}{192} \overset{?}{=} \frac{12}{192}$$

$$\frac{60-48}{192} \overset{?}{=} \frac{12}{192}$$

$$\frac{12}{192} = \frac{12}{192} \checkmark$$

The solution is 14.

9. **CCSS STRUCTURE** Sara has 10 pounds of dried fruit selling for $6.25 per pound. She wants to know how many pounds of mixed nuts selling for $4.50 per pound she needs to make a trail mix selling for $5 per pound.

a. Let m = the number of pounds of mixed nuts. Complete the following table.

	Pounds	Price per Pound	Total Price
Dried Fruit	10	$6.25	6.25(10)
Mixed Nuts			
Trail Mix			

b. Write a rational equation using the last column of the table.

c. Solve the equation to determine how many pounds of mixed nuts are needed.

SOLUTION:

a.

	Pounds	Price per Pound	Total Price
Dried Fruit	10	$6.25	6.25(10)
Mixed Nuts	m	$4.50	4.5m
Trail Mix	10 + m	$5.00	5(10 + m)

b.

$$6.25(10) + 4.5m = 5(10 + m)$$
$$62.5 + 4.5m = 50 + 5m$$

c.

$$62.5 + 4.5m = 50 + 5m$$
$$4.5m - 5m = 50 - 62.5$$
$$-0.5m = -12.5$$
$$m = 25$$

Therefore, 25 pounds of mixed nuts are needed.

10. **DISTANCE** Alicia's average speed riding her bike is 11.5 miles per hour. She takes a round trip of 40 miles. It takes her 1 hour and 20 minutes with the wind and 2 hours and 30 minutes against the wind.

a. Write an expression for Alicia's time with the wind.

b. Write an expression for Alicia's time against the wind.

c. How long does it take to complete the trip?

d. Write and solve the rational equation to determine the speed of the wind.

SOLUTION:

a. Let x be the speed of the wind.
The expression for Alicia's time with the wind is
$$\frac{20}{11.5 + x}.$$

b. The expression for Alicia's time against the wind is $\frac{20}{11.5 - x}.$

c.
1 hr 20 minutes + 2 hr 30 minutes = 3 hr 50 minutes

d.

$$\frac{20}{11.5 + x} + \frac{20}{11.5 - x} = 3\frac{5}{6}$$
$$\frac{20}{11.5 + x} + \frac{20}{11.5 - x} = \frac{23}{6}$$
$$\frac{20(11.5 - x) + 20(11.5 + x)}{(11.5 + x)(11.5 - x)} = \frac{23}{6}$$
$$\frac{230 - 20x + 230 + 20x}{132.25 - x^2} = \frac{23}{6}$$
$$\frac{460}{132.25 - x^2} = \frac{23}{6}$$
$$460(6) = 23(132.25 - x^2)$$
$$2760 = 3041.75 - 23x^2$$
$$23x^2 = 3041.75 - 2760$$
$$23x^2 = 281.75$$
$$x^2 = 12.25$$
$$x = 3.5$$

The speed of the wind is 3.5 mph.

11. **WORK** Kendal and Chandi wax cars. Kendal can wax a particular car in 60 minutes and Chandi can wax the same car in 80 minutes. They plan on waxing the same car together and want to know how long it will take.

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a. How much will Kendal complete in 1 minute?

b. How much will Kendal complete in x minutes?

c. How much will Chandi complete in 1 minute?

d. How much will Chandi complete in x minutes?

e. Write a rational equation representing Kendal and Chandi working together on the car.

f. Solve the equation to determine how long it will take them to finish the ca**r.**

SOLUTION:

a. $\dfrac{1}{60}$

b. $x\left(\dfrac{1}{60}\right) = \dfrac{x}{60}$

c. $\dfrac{1}{80}$

d. $x\left(\dfrac{1}{80}\right) = \dfrac{x}{80}$

e. $\dfrac{x}{60} + \dfrac{x}{80} = 1$

f.

$$\dfrac{x}{60} + \dfrac{x}{80} = 1$$

$$\dfrac{80x + 60x}{4800} = 1$$

$$\dfrac{140x}{4800} = 1$$

$$140x = 4800$$

$$x = \dfrac{4800}{140}$$

$$\approx 34.3$$

It will take them about 34.3 minutes to finish the car.

Solve each inequality. Check your solutions.
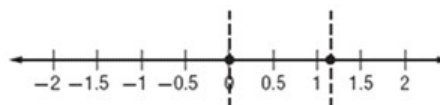
12. $\dfrac{3}{5x} + \dfrac{1}{6x} > \dfrac{2}{3}$

SOLUTION:

The excluded value for this inequality is 0.

Solve the related equation $\dfrac{3}{5x} + \dfrac{1}{6x} = \dfrac{2}{3}$.

$$\dfrac{3}{5x} + \dfrac{1}{6x} = \dfrac{2}{3}$$

$$\dfrac{18x + 5x}{30x^2} = \dfrac{2}{3}$$

$$\dfrac{23x}{30x^2} = \dfrac{2}{3}$$

$$\dfrac{23}{30x} = \dfrac{2}{3}$$

$$x = \dfrac{3(23)}{30(2)}$$

$$x = \dfrac{23}{20}$$

$$x = 1.15$$

Divide the real line in to three intervals as shown.

Test $x = -1$.

$$\dfrac{3}{5(-1)} + \dfrac{1}{6(-1)} \overset{?}{>} \dfrac{2}{3}$$

$$-\dfrac{3}{5} - \dfrac{1}{6} \overset{?}{>} \dfrac{2}{3}$$

$$\dfrac{-18 - 5}{30} \overset{?}{>} \dfrac{2}{3}$$

$$\dfrac{-23}{30} \not> \dfrac{2}{3}$$

Test $x = 1$.

$$\frac{3}{5(1)}+\frac{1}{6(1)}\overset{?}{>}\frac{2}{3}$$

$$\frac{3}{5}+\frac{1}{6}\overset{?}{>}\frac{2}{3}$$

$$\frac{18+5}{30}\overset{?}{>}\frac{2}{3}$$

$$\frac{23}{30}>\frac{2}{3} \checkmark$$

Test $x = 2$.

$$\frac{3}{5(2)}+\frac{1}{6(2)}\overset{?}{>}\frac{2}{3}$$

$$\frac{3}{10}+\frac{1}{12}\overset{?}{>}\frac{2}{3}$$

$$\frac{36+10}{120}\overset{?}{>}\frac{2}{3}$$

$$\frac{46}{120}\overset{?}{>}\frac{2}{3}$$

$$\frac{23}{60}\not>\frac{2}{3}$$

Therefore, the solution is $0 < x < 1.15$.

13. $\dfrac{1}{4c}+\dfrac{1}{9c}<\dfrac{1}{2}$

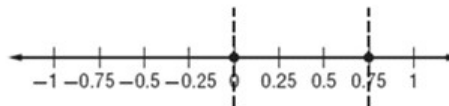*SOLUTION:*

The excluded value for this inequality is 0.

Solve the related equation $\dfrac{1}{4c}+\dfrac{1}{9c}=\dfrac{1}{2}$.

$$\frac{1}{4c}+\frac{1}{9c}=\frac{1}{2}$$

$$\frac{9c+4c}{36c^2}=\frac{1}{2}$$

$$\frac{13c}{36c^2}=\frac{1}{2}$$

$$\frac{13}{36c}=\frac{1}{2}$$

$$c=\frac{26}{36}$$

$$c=\frac{13}{18}$$

Divide the real line in to three intervals as shown.

Test $c = -1$.

$$\frac{1}{4(-1)}+\frac{1}{9(-1)}\overset{?}{<}\frac{1}{2}$$

$$-\frac{1}{4}-\frac{1}{9}\overset{?}{<}\frac{1}{2}$$

$$\frac{-9-4}{36}\overset{?}{<}\frac{1}{2}$$

$$-\frac{13}{36}<\frac{1}{2} \checkmark$$

Test $c = 0.5$.

$$\frac{1}{4(0.5)}+\frac{1}{9(0.5)}\overset{?}{<}\frac{1}{2}$$

$$\frac{1}{2}+\frac{1}{4.5}\overset{?}{<}\frac{1}{2}$$

$$\frac{4.5+2}{9}\overset{?}{<}\frac{1}{2}$$

$$\frac{6.5}{9}\not<\frac{1}{2}$$

Test $c = 1$.

$$\frac{1}{4(1)}+\frac{1}{9(1)}\overset{?}{<}\frac{1}{2}$$

$$\frac{1}{4}+\frac{1}{9}\overset{?}{<}\frac{1}{2}$$

$$\frac{9+4}{36}\overset{?}{<}\frac{1}{2}$$

$$\frac{13}{36}\overset{?}{<}\frac{1}{2}$$

$$\frac{13}{36}<\frac{1}{2}\;\checkmark$$

Therefore, the solution is $c<0$ or $\frac{13}{18}<c$.

14. $\dfrac{4}{3y}+\dfrac{2}{5y}<\dfrac{3}{2}$

SOLUTION:
The excluded value for this inequality is $y=0$.

Solve the related equation $\dfrac{4}{3y}+\dfrac{2}{5y}=\dfrac{3}{2}$.

$$\frac{4}{3y}+\frac{2}{5y}=\frac{3}{2}$$
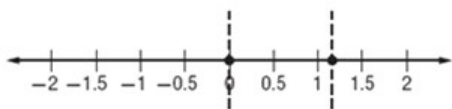
$$\frac{20y+6y}{15y^2}=\frac{3}{2}$$

$$\frac{26y}{15y^2}=\frac{3}{2}$$

$$\frac{26}{15y}=\frac{3}{2}$$

$$y=\frac{52}{45}$$

Divide the real line in to three intervals as shown.

Test $y=-1$.

$$\frac{4}{3(-1)}+\frac{2}{5(-1)}\overset{?}{<}\frac{3}{2}$$

$$-\frac{4}{3}-\frac{2}{5}\overset{?}{<}\frac{3}{2}$$

$$\frac{-20-6}{15}\overset{?}{<}\frac{3}{2}$$

$$-\frac{26}{15}<\frac{3}{2}\;\checkmark$$

Test $y=1$.

$$\frac{4}{3(1)}+\frac{2}{5(1)}\overset{?}{<}\frac{3}{2}$$

$$\frac{4}{3}+\frac{2}{5}\overset{?}{<}\frac{3}{2}$$

$$\frac{20+6}{15}\overset{?}{<}\frac{3}{2}$$

$$\frac{26}{15}\not<\frac{3}{2}$$

Test $y=2$.

$$\frac{4}{3(2)}+\frac{2}{5(2)}\overset{?}{<}\frac{3}{2}$$

$$\frac{4}{6}+\frac{2}{10}\overset{?}{<}\frac{3}{2}$$

$$\frac{2}{3}+\frac{1}{5}\overset{?}{<}\frac{3}{2}$$

$$\frac{10+3}{15}\overset{?}{<}\frac{3}{2}$$

$$\frac{13}{15}\overset{?}{<}\frac{3}{2}$$

$$\frac{13}{15}<\frac{3}{2}\;\checkmark$$

Therefore, the solution is $y<0$ or $y>\dfrac{52}{45}$.

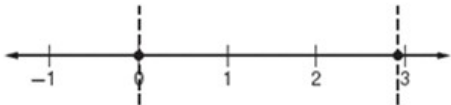15. $\dfrac{1}{3b}+\dfrac{1}{4b}<\dfrac{1}{5}$

SOLUTION:
The excluded value of this inequality is $b=0$.

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Solve the related equation $\dfrac{1}{3b}+\dfrac{1}{4b}=\dfrac{1}{5}$.

$$\dfrac{1}{3b}+\dfrac{1}{4b}=\dfrac{1}{5}$$

$$\dfrac{4b+3b}{12b^2}=\dfrac{1}{5}$$

$$\dfrac{7b}{12b^2}=\dfrac{1}{5}$$

$$\dfrac{7}{12b}=\dfrac{1}{5}$$

$$12b=35$$

$$b=\dfrac{35}{12}$$

Divide the real line in to three intervals as shown.

Test $b=-1$.

$$\dfrac{1}{3(-1)}+\dfrac{1}{4(-1)}\overset{?}{<}\dfrac{1}{5}$$

$$-\dfrac{1}{3}-\dfrac{1}{4}\overset{?}{<}\dfrac{1}{5}$$

$$\dfrac{-4-3}{12}\overset{?}{<}\dfrac{1}{5}$$

$$-\dfrac{7}{12}<\dfrac{1}{5}\checkmark$$

Test $b=1$.

$$\dfrac{1}{3}+\dfrac{1}{4}\overset{?}{<}\dfrac{1}{5}$$

$$\dfrac{4+3}{12}\overset{?}{<}\dfrac{1}{5}$$

$$\dfrac{7}{12}\overset{?}{<}\dfrac{1}{5}$$

$$\dfrac{7}{12}\not<\dfrac{1}{5}$$

Test $b=3$.

$$\dfrac{1}{9}+\dfrac{1}{12}\overset{?}{<}\dfrac{1}{5}$$

$$\dfrac{12+9}{108}\overset{?}{<}\dfrac{1}{5}$$

$$\dfrac{21}{108}\overset{?}{<}\dfrac{1}{5}$$

$$\dfrac{21}{108}<\dfrac{1}{5}\checkmark$$

Therefore, the solution is $b<0$, or $\dfrac{35}{12}<b$.

Solve each equation. Check your solutions.

16. $\dfrac{9}{x-7}-\dfrac{7}{x-6}=\dfrac{13}{x^2-13x+42}$

SOLUTION:

$$\dfrac{9}{x-7}-\dfrac{7}{x-6}=\dfrac{13}{x^2-13x+42}$$

$$\dfrac{9(x-6)-7(x-7)}{(x-7)(x-6)}=\dfrac{13}{x^2-13x+42}$$

$$\dfrac{9x-54-7x+49}{x^2-13x+42}=\dfrac{13}{x^2-13x+42}$$

$$2x-5=13$$

$$2x=18$$

$$x=9$$

Check:

$$\dfrac{9}{9-7}-\dfrac{7}{9-6}\overset{?}{=}\dfrac{13}{9^2-13(9)+42}$$

$$\dfrac{9}{2}-\dfrac{7}{3}\overset{?}{=}\dfrac{13}{81-117+42}$$

$$\dfrac{27-14}{6}\overset{?}{=}\dfrac{13}{6}$$

$$\dfrac{13}{6}=\dfrac{13}{6}\checkmark$$

The solution is 9.

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17. $\dfrac{13}{y+3} - \dfrac{12}{y+4} = \dfrac{18}{y^2+7y+12}$

SOLUTION:

$$\dfrac{13}{y+3} - \dfrac{12}{y+4} = \dfrac{18}{y^2+7y+12}$$

$$\dfrac{13(y+4)-12(y+3)}{y^2+3y+4y+12} = \dfrac{18}{y^2+7y+12}$$

$$\dfrac{13y+52-12y-36}{y^2+7y+12} = \dfrac{18}{y^2+7y+12}$$

$$y+16=18$$

$$y=2$$

Check:

$$\dfrac{13}{2+3} - \dfrac{12}{2+4} \overset{?}{=} \dfrac{18}{2^2+7(2)+12}$$

$$\dfrac{13}{5} - \dfrac{12}{6} \overset{?}{=} \dfrac{18}{4+14+12}$$

$$\dfrac{78-60}{30} \overset{?}{=} \dfrac{18}{30}$$

$$\dfrac{18}{30} = \dfrac{18}{30} \checkmark$$

The solution is 2.

18. $\dfrac{14}{x-2} - \dfrac{18}{x+1} = \dfrac{22}{x^2-x-2}$

SOLUTION:

$$\dfrac{14}{x-2} - \dfrac{18}{x+1} = \dfrac{22}{x^2-x-2}$$

$$\dfrac{14(x+1)-18(x-2)}{(x-2)(x+1)} = \dfrac{22}{x^2-x-2}$$

$$\dfrac{14x+14-18x+36}{x^2-2x+x-2} = \dfrac{22}{x^2-x-2}$$

$$\dfrac{-4x+50}{x^2-x-2} = \dfrac{22}{x^2-x-2}$$

$$-4x+50=22$$

$$-4x=-28$$

$$x=7$$

Check:

$$\dfrac{14}{7-2} - \dfrac{18}{7+1} \overset{?}{=} \dfrac{22}{7^2-7-2}$$

$$\dfrac{14}{5} - \dfrac{18}{8} \overset{?}{=} \dfrac{22}{49-7-2}$$

$$\dfrac{112-90}{40} \overset{?}{=} \dfrac{22}{40}$$

$$\dfrac{22}{40} = \dfrac{22}{40} \checkmark$$

The solution is 7.

19. $\dfrac{11}{a+2} - \dfrac{10}{a+5} = \dfrac{36}{a^2+7a+10}$

SOLUTION:

$$\dfrac{11}{a+2} - \dfrac{10}{a+5} = \dfrac{36}{a^2+7a+10}$$

$$\dfrac{11(a+5)-10(a+2)}{(a+2)(a+5)} = \dfrac{36}{a^2+7a+10}$$

$$\dfrac{11a+55-10a-20}{a^2+2a+5a+10} = \dfrac{36}{a^2+7a+10}$$

$$\dfrac{a+35}{a^2+7a+10} = \dfrac{36}{a^2+7a+10}$$

$$a+35 = 36$$

$$a = 1$$

Check:

$$\dfrac{11}{1+2} - \dfrac{10}{1+5} \overset{?}{=} \dfrac{36}{1^2+7+10}$$

$$\dfrac{11}{3} - \dfrac{10}{6} \overset{?}{=} \dfrac{36}{1+7+10}$$

$$\dfrac{22-10}{6} \overset{?}{=} \dfrac{36}{18}$$

$$\dfrac{12}{6} \overset{?}{=} 2$$

$$2 = 2 \checkmark$$

The solution is 1.

20. $\dfrac{x}{2x-1} + \dfrac{3}{x+4} = \dfrac{21}{2x^2+7x-4}$

SOLUTION:

$$\dfrac{x}{2x-1} + \dfrac{3}{x+4} = \dfrac{21}{2x^2+7x-4}$$

$$\dfrac{x}{2x-1} + \dfrac{3}{x+4} = \dfrac{21}{(2x-1)(x+4)}$$

$$\dfrac{x(x+4)}{(2x-1)(x+4)} + \dfrac{3(2x-1)}{(x+4)(2x-1)} = \dfrac{21}{(2x-1)(x+4)}$$

$$\dfrac{x^2+4x}{(2x-1)(x+4)} + \dfrac{6x-3}{(x+4)(2x-1)} = \dfrac{21}{(2x-1)(x+4)}$$

$$\dfrac{x^2+4x+6x-3}{(2x-1)(x+4)} = \dfrac{21}{(2x-1)(x+4)}$$

$$x^2+4x+6x-3 = 21$$

$$x^2+10x-3 = 21$$

Use the quadratic formula.

$$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \dfrac{-10 \pm \sqrt{10^2-4(1)(-24)}}{2(1)}$$

$$= \dfrac{-10 \pm \sqrt{196}}{2}$$

$$= \dfrac{-10 \pm 14}{2}$$

$$= -5 \pm 7$$

$$= 2 \text{ or } -12$$

Check: $x = 2$

$$\dfrac{2}{2(2)-1} + \dfrac{3}{2+4} \overset{?}{=} \dfrac{21}{2(2)^2+7(2)-4}$$

$$\dfrac{2}{3} + \dfrac{3}{6} \overset{?}{=} \dfrac{21}{8+14-4}$$

$$1.17 \approx 1.17 \checkmark$$

Check: $x = -12$

$$\dfrac{-12}{2(-12)-1} + \dfrac{3}{-12+4} \overset{?}{=} \dfrac{21}{2(-12)^2+7(-12)-4}$$

$$\dfrac{-12}{-25} + \dfrac{3}{-8} \overset{?}{=} \dfrac{21}{288-84-4}$$

$$\dfrac{-12}{-25} + \dfrac{3}{-8} \overset{?}{=} \dfrac{21}{200}$$

$$0.105 \approx 0.105 \checkmark$$

Therefore, the solution set is $\{2, -12\}$

21. $\dfrac{2}{y-5} + \dfrac{y-1}{2y+1} = \dfrac{2}{2y^2-9y-5}$

SOLUTION:

$$\dfrac{2}{y-5} + \dfrac{y-1}{2y+1} = \dfrac{2}{2y^2-9y-5}$$

$$\dfrac{2(2y+1)+(y-1)(y-5)}{(y-5)(2y+1)} = \dfrac{2}{2y^2-9y-5}$$

$$\dfrac{4y+2+y^2-5y-y+5}{2y^2+y-10y-5} = \dfrac{2}{2y^2-9y-5}$$

$$\dfrac{y^2-2y+7}{2y^2-9y-5} = \dfrac{2}{2y^2-9y-5}$$

$$y^2-2y+7 = 2$$

$$y^2-2y+7-2 = 0$$

$$y^2-2y+5 = 0$$

Use the Quadratic formula to solve $y^2-2y+5=0$.

$$y = \dfrac{2 \pm \sqrt{4-20}}{2} \quad \text{(imaginary)}$$

There is no real solution for the quadratic equation $y^2-2y+5=0$. Therefore, the solution for the given rational equation is $\varnothing$.

22. **CHEMISTRY** How many milliliters of a 20% acid solution must be added to 40 milliliters of a 75% acid solution to create a 30% acid solution?

SOLUTION:
Let x milliliters of a 20% acid solution is added to 40 milliliters of a 75% acid solution.

	Original	Added	New
Amount of Acid	0.75(40)	0.2(x)	0.75(40) + 0.2(x)
Total Solution	40	x	40 + x

$$\dfrac{\text{percent}}{100} = \dfrac{\text{amount of acid}}{\text{total solution}}$$

$$\dfrac{30}{100} = \dfrac{0.75(40)+0.2(x)}{40+x}$$

$$\dfrac{3}{10} = \dfrac{7.5(4)+0.2(x)}{40+x}$$

$$3(40+x) = 10\left[7.5(4)+0.2(x)\right]$$

$$120+3x = 300+2x$$

$$x = 300-120$$

$$x = 180$$

Check:

$$\dfrac{30}{100} \overset{?}{=} \dfrac{0.75(40)+0.2(180)}{40+180}$$

$$\dfrac{30}{100} \overset{?}{=} \dfrac{30+36}{220}$$

$$\dfrac{30}{100} \overset{?}{=} \dfrac{66}{220}$$

$$0.3 = 0.3 \checkmark$$

Therefore, 180 milliliters of a 20% acid solution must be added to 40 milliliters of a 75% acid solution to create a 30% acid solution.

23. **GROCERIES** Ellen bought 3 pounds of bananas for $0.90 per pound. How many pounds of apples costing $1.25 per pound must she purchase so that the total cost for fruit is $1 per pound?

SOLUTION:

Let Ellen bought x pounds of apples.

$$3(0.90) + x(1.25) = (3 + x)(1)$$
$$2.7 + 1.25x = 3 + x$$
$$1.25x - x = 3 - 2.7$$
$$0.25x = 0.3$$
$$x = \frac{0.3}{0.25}$$
$$x = 1.2$$

She needs to purchase 1.2 pounds of apples.

24. **BUILDING** Bryan's volunteer group can build a garage in 12 hours. Sequoia's group can build it in 16 hours. How long would it take them if they worked together?

SOLUTION:

The rate for Bryan's volunteer group is $\dfrac{1}{12}$.

The rate for Sequoia's group is $\dfrac{1}{16}$.

Let their combined rate is $\dfrac{1}{x}$.

$$\frac{1}{12} + \frac{1}{16} = \frac{1}{x}$$
$$\frac{16 + 12}{192} = \frac{1}{x}$$
$$\frac{28}{192} = \frac{1}{x}$$
$$28x = 192$$
$$x = \frac{192}{28}$$
$$x \approx 6.86$$

Therefore, it would take about 6.86 hours to build a garage if they worked together.

Solve each inequality. Check your solutions.

25. $3 - \dfrac{4}{x} > \dfrac{5}{4x}$

SOLUTION:

The excluded value for this inequality is $x = 0$.

$$3 - \frac{4}{x} > \frac{5}{4x}$$

$$\frac{3x-4}{x} > \frac{5}{4x}$$

$$3x - 4 > \frac{5}{4}$$

$$4(3x - 4) > 5$$
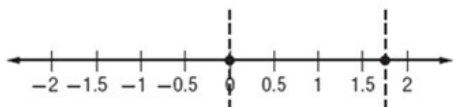
$$12x - 16 > 5$$

$$12x > 21$$

$$x > \frac{21}{12}$$

$$x > \frac{7}{4}$$

$$x > 1.75$$

Divide the real line in to three intervals as shown.

Test $x = -1$.

$$3 - \frac{4}{(-1)} \overset{?}{>} \frac{5}{4(-1)}$$

$$3 + 4 \overset{?}{>} -\frac{5}{4}$$

$$7 > -\frac{5}{4} \checkmark$$

Test $x = 1$.

$$3 - \frac{4}{(1)} \overset{?}{>} \frac{5}{4(1)}$$

$$3 - 4 \overset{?}{>} \frac{5}{4}$$

$$-1 \not> -\frac{5}{4}$$

Test $x = 2$.

$$3 - \frac{4}{2} \overset{?}{>} \frac{5}{4(2)}$$

$$\frac{6-4}{2} \overset{?}{>} \frac{5}{8}$$

$$1 > \frac{5}{8} \checkmark$$

The solution for the inequality is $x < 0$ or $x > 1.75$.

26. $\dfrac{5}{3a} - \dfrac{3}{4a} > \dfrac{5}{6}$

SOLUTION:
The excluded value for this inequality is $a = 0$.

$$\frac{5}{3a} - \frac{3}{4a} > \frac{5}{6}$$

$$\frac{20a - 9a}{12a^2} > \frac{5}{6}$$

$$\frac{11a}{12a^2} > \frac{5}{6}$$

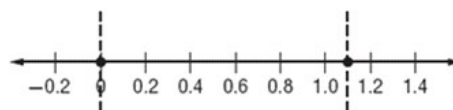$$\frac{11}{12a} > \frac{5}{6}$$

$$66 > 60a$$

$$60a < 66$$

$$a < \frac{66}{60}$$

$$a < \frac{11}{10}$$

$$a < 1.1$$

Divide the inequality in to three intervals as shown.

Test $a = -1$.

$$\frac{5}{-3} - \frac{3}{4(-1)} \overset{?}{>} \frac{5}{6}$$

$$-\frac{5}{3} + \frac{3}{4} \overset{?}{>} \frac{5}{6}$$

$$\frac{-20 + 9}{12} \overset{?}{>} \frac{5}{6}$$

$$-\frac{11}{12} \not> \frac{5}{6}$$

Test $a = 1$.

$$\frac{5}{3}-\frac{3}{4}\overset{?}{>}\frac{5}{6}$$

$$\frac{20-9}{12}\overset{?}{>}\frac{5}{6}$$

$$\frac{11}{12}>\frac{5}{6}$$

Test $a = 2$.

$$\frac{5}{6}-\frac{3}{8}\overset{?}{>}\frac{5}{6}$$

$$\frac{20-9}{24}\overset{?}{>}\frac{5}{6}$$

$$\frac{11}{24}\not>\frac{5}{6}$$

Therefore, the solution set is $0 < a < 1.1$.

27. $\dfrac{x-2}{x+2}+\dfrac{1}{x-2}>\dfrac{x-4}{x-2}$

SOLUTION:

The excluded values for this inequality is $x = -2$ and $x = 2$.

Solve the related equation $\dfrac{x-2}{x+2}+\dfrac{1}{x-2}=\dfrac{x-4}{x-2}$.

$$\frac{x-2}{x+2}+\frac{1}{x-2}=\frac{x-4}{x-2}$$

$$\frac{(x-2)(x-2)+(x+2)}{(x+2)(x-2)}=\frac{x-4}{x-2}$$

$$\frac{x^2-4x+4+x+2}{(x+2)}=x-4$$

$$x^2-3x+6=(x-4)(x+2)$$

$$x^2-3x+6=x^2+2x-4x-8$$
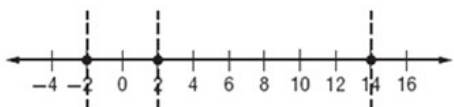
$$x^2-3x+6=x^2-2x-8$$

$$-x=-8-6$$

$$-x=-14$$

$$x=14$$

Divide the real line in to four intervals as shown.

Test $x = -4$.

$$\frac{-4-2}{-4+2}+\frac{1}{-4-2}\overset{?}{>}\frac{-4-4}{-4-2}$$

$$\frac{-6}{-2}-\frac{1}{6}\overset{?}{>}\frac{-8}{-6}$$

$$3-\frac{1}{6}\overset{?}{>}\frac{4}{3}$$

$$\frac{18-1}{6}\overset{?}{>}\frac{4}{3}$$

$$\frac{17}{6}\overset{?}{>}\frac{4}{3}$$

$$\frac{17}{6}>\frac{4}{3}\checkmark$$

Test $x = 0$.

$$\frac{-2}{2}+\frac{1}{-2}\overset{?}{>}\frac{-4}{-2}$$

$$-1-\frac{1}{2}\overset{?}{>}2$$

$$\frac{-2-1}{2}\overset{?}{>}2$$

$$-\frac{3}{2}\overset{?}{>}2$$

$$-1.5\not>2$$

Test $x = 4$.

$$\frac{4-2}{4+2}+\frac{1}{4-2}\overset{?}{>}\frac{4-4}{4-2}$$

$$\frac{2}{6}+\frac{1}{2}\overset{?}{>}0$$

$$\frac{2+3}{6}\overset{?}{>}0$$

$$\frac{5}{6}>0\checkmark$$

Test $x = 16$.

$$\frac{16-2}{16+2}+\frac{1}{16-2}\overset{?}{>}\frac{16-4}{16-2}$$

$$\frac{14}{18}+\frac{1}{14}\overset{?}{>}\frac{12}{14}$$

$$\frac{7}{9}+\frac{1}{14}\overset{?}{>}\frac{6}{7}$$

$$\frac{98+9}{126}\overset{?}{>}\frac{6}{7}$$

$$\frac{107}{126}\not>\frac{6}{7}$$

Therefore, the solution set for the inequality is $x < -2$ or $2 < x < 14$.

28. $\dfrac{3}{4}-\dfrac{1}{x-3}>\dfrac{x}{x+4}$

SOLUTION:

The excluded value for this inequality is $x = 3$ and $x = -4$.

Solve the related equation $\dfrac{3}{4}-\dfrac{1}{x-3}=\dfrac{x}{x+4}$.

$$\frac{3}{4}-\frac{1}{x-3}=\frac{x}{x+4}$$

$$\frac{3(x-3)-4}{4(x-3)}=\frac{x}{x+4}$$

$$\frac{3x-9-4}{4x-12}=\frac{x}{x+4}$$

$$\frac{3x-13}{4x-12}=\frac{x}{x+4}$$

$$(3x-13)(x+4)=x(4x-12)$$

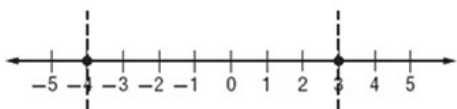$$3x^2+12x-13x-52=4x^2-12x$$

$$-x^2+11x-52=0$$

$$x^2-11x+52=0$$

There exists no real solution for the quadratic equation $x^2-11x+52=0$.
Divide the real line in to three intervals as shown.

Test $x = -5$.

$$\frac{3}{4}-\frac{1}{-5-3}\overset{?}{>}\frac{-5}{-5+4}$$

$$\frac{3}{4}-\frac{1}{-8}\overset{?}{>}\frac{-5}{-1}$$

$$\frac{3}{4}+\frac{1}{8}\overset{?}{>}5$$

$$\frac{6+1}{8}\overset{?}{>}5$$

$$\frac{7}{8}\not>5$$

Test $x = 0$.

$$\frac{3}{4}-\frac{1}{0-3}\overset{?}{>}\frac{0}{0+4}$$

$$\frac{3}{4}+\frac{1}{3}\overset{?}{>}0$$

$$\frac{9+4}{12}\overset{?}{>}0$$

$$\frac{13}{12}>0\checkmark$$

Test $x = 5$.

$$\frac{3}{4}-\frac{1}{5-3}\overset{?}{>}\frac{5}{5+4}$$

$$\frac{3}{4}-\frac{1}{2}\overset{?}{>}\frac{5}{9}$$

$$\frac{3-2}{4}\overset{?}{>}\frac{5}{9}$$

$$\frac{1}{4}\not>\frac{5}{9}$$

The solution set is $-4 < x < 3$.

29. $\dfrac{x}{5}+\dfrac{2}{3}<\dfrac{3}{x-4}$

SOLUTION:

The excluded value for this inequality is $x = 4$.

Solve the related equation $\dfrac{x}{5}+\dfrac{2}{3}=\dfrac{3}{x-4}$.

8-6 Solving Rational Equations and Inequalities

$$\frac{x}{5}+\frac{2}{3}=\frac{3}{x-4}$$

$$\frac{3x+10}{15}=\frac{3}{x-4}$$

$$(3x+10)(x-4)=45$$

$$3x^2-12x+10x-40=45$$

$$3x^2-2x-40-45=0$$

$$3x^2-2x-85=0$$

Solve the quadratic equation using the Quadratic formula.

$$x=\frac{2\pm\sqrt{4-4(3)(-85)}}{6}$$

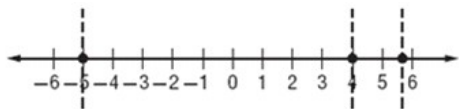$$=\frac{2\pm\sqrt{4+1020}}{6}$$

$$=\frac{2\pm\sqrt{1024}}{6}$$

$$=\frac{2\pm32}{6}$$

$$x=\frac{2+32}{6}\quad\text{or}\quad x=\frac{2-32}{6}$$

$$x=\frac{34}{6}\quad\text{or}\quad x=-\frac{30}{6}$$

$$x=\frac{17}{3}\quad\text{or}\quad x=-5$$

Divide the real line in to 4 intervals as shown.

Test $x=-6$.

$$\frac{-6}{5}+\frac{2}{3}\overset{?}{<}\frac{3}{-6-4}$$

$$-\frac{6}{5}+\frac{2}{3}\overset{?}{<}-\frac{3}{10}$$

$$\frac{-18+10}{15}\overset{?}{<}-\frac{3}{10}$$

$$-\frac{8}{15}<-\frac{3}{10}\ \checkmark$$

Test $x=0$.

$$\frac{0}{5}+\frac{2}{3}\overset{?}{<}\frac{3}{0-4}$$

$$\frac{2}{3}\not<-\frac{3}{4}$$

Test $x=5$.

$$\frac{5}{5}+\frac{2}{3}\overset{?}{<}\frac{3}{5-4}$$

$$1+\frac{2}{3}\overset{?}{<}\frac{3}{1}$$

$$\frac{3+2}{3}\overset{?}{<}3$$

$$\frac{5}{3}<3\ \checkmark$$

Test $x=6$.

$$\frac{6}{5}+\frac{2}{3}\overset{?}{<}\frac{3}{6-4}$$

$$\frac{6}{5}+\frac{2}{3}\overset{?}{<}\frac{3}{3}$$

$$\frac{18+10}{15}\overset{?}{<}1$$

$$\frac{28}{15}\not<1$$

The solution set for the inequality is $x<-5$ or

$$4<x<\frac{17}{3}.$$

30. $\dfrac{x}{x+2}+\dfrac{1}{x-1}<\dfrac{3}{2}$

SOLUTION:
The excluded values for this inequality are $x=-2$ and $x=1$.

Solve the related equation $\dfrac{x}{x+2}+\dfrac{1}{x-1}=\dfrac{3}{2}$.

$$\frac{x}{x+2}+\frac{1}{x-1}=\frac{3}{2}$$

$$\frac{x(x-1)+(x+2)}{(x+2)(x-1)}=\frac{3}{2}$$

$$\frac{x^2-x+x+2}{x^2-x+2x-2}=\frac{3}{2}$$

$$\frac{x^2+2}{x^2+x-2}=\frac{3}{2}$$

$$2(x^2+2)=3(x^2+x-2)$$

$$2x^2+4=3x^2+3x-6$$

$$-x^2-3x+10=0$$

$$x^2+3x-10=0$$

$$(x+5)(x-2)=0$$

$$x+5=0 \text{ or } x-2=0$$

$$x=-5 \text{ or } x=2$$

Divide the real line in to 5 intervals as shown.

Test $x = -6$.

$$\frac{-6}{-6+2}+\frac{1}{-6-1}\overset{?}{<}\frac{3}{2}$$

$$\frac{-6}{-4}-\frac{1}{7}\overset{?}{<}\frac{3}{2}$$

$$\frac{3}{2}-\frac{1}{7}\overset{?}{<}\frac{3}{2}$$

$$\frac{21-2}{14}\overset{?}{<}\frac{3}{2}$$

$$\frac{19}{14}<\frac{3}{2}\checkmark$$

Test $x = -4$.

$$\frac{-4}{-4+2}+\frac{1}{-4-1}\overset{?}{<}\frac{3}{2}$$

$$\frac{-4}{-2}-\frac{1}{5}\overset{?}{<}\frac{3}{2}$$

$$2-\frac{1}{5}\overset{?}{<}\frac{3}{2}$$

$$\frac{10-1}{5}\overset{?}{<}\frac{3}{2}$$

$$\frac{9}{5}\not<\frac{3}{2}$$

Test $x = 0$.

$$\frac{0}{0+2}+\frac{1}{0-1}\overset{?}{<}\frac{3}{2}$$

$$-1<\frac{3}{2}\checkmark$$

Test $x = \frac{3}{2}$.

$$\frac{\left(\frac{3}{2}\right)}{\frac{3}{2}+2}+\frac{1}{\frac{3}{2}-1}\overset{?}{<}\frac{3}{2}$$

$$\frac{\left(\frac{3}{2}\right)}{\left(\frac{7}{3}\right)}+\frac{1}{\left(\frac{1}{2}\right)}\overset{?}{<}\frac{3}{2}$$

$$\frac{3}{2}\cdot\frac{3}{7}+2\overset{?}{<}\frac{3}{2}$$

$$\frac{9}{14}+2\overset{?}{<}\frac{3}{2}$$

$$\frac{9+28}{14}\overset{?}{<}\frac{3}{2}$$

$$\frac{37}{14}\overset{?}{<}\frac{3}{2}$$

$$2\not<\frac{3}{2}$$

Test $x = 4$.

$$\frac{4}{4+2}+\frac{1}{4-1}\overset{?}{<}\frac{3}{2}$$

$$\frac{4}{6}+\frac{1}{3}\overset{?}{<}\frac{3}{2}$$

$$\frac{2}{3}+\frac{1}{3}\overset{?}{<}\frac{3}{2}$$

$$1<\frac{3}{2}\checkmark$$

The solution set for the inequality is $x < -5$ or $-2 < x < 1$ or $x > 2$.

31. **AIR TRAVEL** It takes a plane 20 hours to fly to its destination against the wind. The return trip takes 16 hours. If the plane's average speed in still air is 500 miles per hour, what is the average speed of the wind during the flight?

SOLUTION:

Distance traveled with the wind	Distance traveled against the wind
$(500+w)(16)$	$(500-w)(20)$

$$(500+w)(16)=(500-w)(20)$$
$$(500+w)(4)=(500-w)(5)$$
$$2000+4w=2500-5w$$
$$9w=500$$
$$w=\frac{500}{9}$$
$$w\approx 55.56$$

The average speed of the wind during the flight is about 55.56 miles per hour.

32. **FINANCIAL LITERACY** Judie wants to invest $10,000 in two different accounts. The risky account earns 9% interest, while the other account earns 5% interest. She wants to earn $750 interest for the year. Of tables, graphs, or equations, choose the best representation needed and determine how much should be invested in each account.

SOLUTION:

Judie invest x dollars in the account earns 9% interest and $(10000 - x)$ dollars in the account earns 5% interest.

$$0.09x+0.05(10000-x)=750$$
$$0.09x+500-0.05x=750$$
$$0.04x=250$$
$$x=\frac{250}{0.04}$$
$$x=6250$$

Thus, Judie should invest $6250 at 9% account and $3750 at 5% account.

33. **MULTIPLE REPRESENTATIONS** Consider $\dfrac{2}{x-3}+\dfrac{1}{x}=\dfrac{x-1}{x-3}$.

a. ALGEBRAIC Solve the equation for x. Were any values of x extraneous?

b. GRAPHICAL Graph $y_1=\dfrac{2}{x-3}+\dfrac{1}{x}$ and $y_2=\dfrac{x-1}{x-3}$ on the same graph for $0 < x < 5$.

c. ANALYTICAL For what value(s) of x do they intersect? Do they intersect where x is extraneous for the original equation?

d. VERBAL Use this knowledge to describe how you can use a graph to determine whether an apparent solution of a rational equation is extraneous.

SOLUTION:
a.

$$\frac{2}{x-3}+\frac{1}{x}=\frac{x-1}{x-3}$$

$$\frac{2x+x-3}{x(x-3)}=\frac{x-1}{x-3}$$

$$\frac{3x-3}{x(x-3)}=\frac{x-1}{x-3}$$

$$3x-3=x(x-1)$$

$$3x-3=x^2-x$$

$$x^2-4x+3=0$$

$$(x-3)(x-1)=0$$

$$x=3 \text{ or } x=1$$

Check: $x = 1$
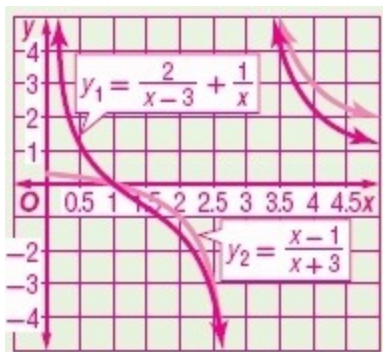
$$\frac{2}{1-3}+\frac{1}{1}\overset{?}{=}\frac{1-1}{1-3}$$

$$-1+1\overset{?}{=}0$$

$$0=0\checkmark$$

$x = 3$ is the excluded value for the equation. Therefore, $x = 3$ is the extraneous solution and $x = 1$ is the solution for the equation.

b.

c. Two graphs intersect at $x = 1$ and they do not intersect at the extraneous solution $x = 3$.]

d. Graph both sides of the equation. Where the graphs intersect, there is a solution. If they do not, then the possible solution is extraneous.

Solve each equation. Check your solutions.

34. $\dfrac{2}{y+3}-\dfrac{3}{4-y}=\dfrac{2y-2}{y^2-y-12}$

SOLUTION:

$$\frac{2}{y+3}-\frac{3}{4-y}=\frac{2y-2}{y^2-y-12}$$

$$\frac{2}{y+3}+\frac{3}{y-4}=\frac{2y-2}{y^2-y-12}$$

$$\frac{2(y-4)+3(y+3)}{(y+3)(y-4)}=\frac{2y-2}{y^2-y-12}$$

$$\frac{2y-8+3y+9}{y^2-y-12}=\frac{2y-2}{y^2-y-12}$$

$$5y+1=2y-2$$

$$3y=-3$$

$$y=-1$$

Check:

$$\frac{2}{2}-\frac{3}{5}\overset{?}{=}\frac{-2-2}{1+1-12}$$

$$1-\frac{3}{5}\overset{?}{=}\frac{-4}{-10}$$

$$\frac{5-3}{5}\overset{?}{=}\frac{2}{5}$$

$$\frac{2}{5}=\frac{2}{5}\checkmark$$

The solution is $y = -1$.

35. $\dfrac{2}{y+2} - \dfrac{y}{2-y} = \dfrac{y^2+4}{y^2-4}$

SOLUTION:

$$\dfrac{2}{y+2} - \dfrac{y}{2-y} = \dfrac{y^2+4}{y^2-4}$$

$$\dfrac{2}{y+2} + \dfrac{y}{y-2} = \dfrac{y^2+4}{y^2-4}$$

$$\dfrac{2(y+2)+y(y+2)}{(y+2)(y-2)} = \dfrac{y^2+4}{y^2-4}$$

$$\dfrac{2y+4+y^2+2y}{y^2-4} = \dfrac{y^2+4}{y^2-4}$$

$$y^2+4y+4 = y^2+4$$

$$4y = 0$$

$$y = 0$$

Check:

$$\dfrac{2}{2} - 0 \overset{?}{=} \dfrac{4}{-4}$$

$$1 \neq -1$$

The solution set is $\varnothing$.

36. **OPEN ENDED** Give an example of a rational equation that can be solved by multiplying each side of the equation by $4(x + 3)(x - 4)$.

SOLUTION:
Sample answer:

$$\dfrac{4}{x+3} = \dfrac{x}{x-4} + \dfrac{7}{4}$$

37. **CHALLENGE** Solve $\dfrac{1+\dfrac{9}{x}+\dfrac{20}{x^2}}{1-\dfrac{25}{x^2}} = \dfrac{x+4}{x-5}$.

SOLUTION:

$$\dfrac{1+\dfrac{9}{x}+\dfrac{20}{x^2}}{1-\dfrac{25}{x^2}} = \dfrac{x+4}{x-5}$$

$$\dfrac{\dfrac{x^2+9x+20}{x^2}}{\dfrac{x^2-25}{x^2}} = \dfrac{x+4}{x-5}$$

$$\dfrac{x^2+9x+20}{x^2-25} = \dfrac{x+4}{x-5}$$

$$\dfrac{x^2+9x+20}{(x+5)(x-5)} = \dfrac{x+4}{x-5}$$

$$\dfrac{x^2+9x+20}{x+5} = x+4$$

$$x^2+9x+20 = (x+4)(x+5)$$

$$x^2+9x+20 = x^2+9x+20$$

Therefore, the solution is all real numbers except 5, −5, and 0.

38. **CCSS TOOLS** While using the table feature on the graphing calculator to explore $f(x) = \dfrac{1}{x^2-x-6}$, the values −2 and 3 say "**ERROR**." Explain its meaning.

SOLUTION:
Sample answer:

$$f(x) \;=\; \dfrac{1}{x^2-x-6}$$

$$\;=\; \dfrac{1}{(x-3)(x+2)}$$

The denominator will equal 0 when $x = -2$ or $x = 3$. The values −2 and 3 are undefined values. On the graph of $f(x)$ there would be vertical asymptotes at these values.

39. **WRITING IN MATH** Why should you check solutions of rational equations and inequalities?

SOLUTION:
Sample answer: Multiplying each side of a rational equation or inequality by the LCD can result in extraneous solutions. Therefore, you should check all solutions to make sure that they satisfy the original equation or inequality.

40. Nine pounds of mixed nuts containing 55% peanuts were mixed with 6 pounds of another kind of mixed nuts that contain 40% peanuts. What percent of the new mixture is peanuts?

   A 58%

   B 51%

   C 49%

   D 47%

SOLUTION:
Let the new mixture contains x percent of peanuts.

$$0.55(9)+0.4(6)=x(9+6)$$
$$4.95+2.4=15x$$
$$7.35=15x$$
$$x=\frac{7.35}{15}$$
$$x=0.49$$

So, the new mixture contains 0.49 or 49% percent of peanuts. The correct choice is C.

41. Working alone, Dato can dig a 10-foot by 10-foot hole in five hours. Pedro can dig the same hole in six hours. How long would it take them if they worked together?

   F 1.5 hours

   G 2.34 hours

   H 2.52 hours

   J 2.73 hours

SOLUTION:
$$\frac{1}{5}+\frac{1}{6}=\frac{1}{x}$$
$$\frac{6+5}{30}=\frac{1}{x}$$
$$\frac{11}{30}=\frac{1}{x}$$
$$x=\frac{30}{11}$$
$$x\approx 2.73$$

It would take about 2.73 hours to dig the hole if they worked together. The correct choice is J.

42. An aircraft carrier made a trip to Guam and back. The trip there took three hours and the trip back took four hours. It averaged 6 kilometers per hour on the return trip. Find the average speed of the trip to Guam.

A 6 km/h

B 8 km/h

C 10 km/h

D 12 km/h

SOLUTION:

$$Speed = \frac{distance}{time}$$

$$6 = \frac{x}{4}$$

$$x = 24$$

Therefore, the distance of the trip to Guam is 24 kilometers.

$$Speed = \frac{distance}{time}$$

$$= \frac{24}{3}$$

$$= 8$$

The average speed of the trip to Guam is 8 km/h. So, the correct choice is B.

43. **SHORT RESPONSE** If a line ℓ is perpendicular to a segment CD at point F and $CF = FD$, how many points on line ℓ are the same distance from point C as from point D?

SOLUTION:
all of the points

Determine whether each relation shows *direct* or *inverse* variation, or *neither*.

x	y
14	3
28	1.5
56	0.75
112	0.375

44.

SOLUTION:

x	y	xy
14	3	42
28	1.5	42
56	0.75	42
112	0.375	42

Since $xy = 42$ (a constant) for all x and y values in the relation, the relation is an inverse variation.

x	y
0.2	24
0.6	72
1.8	216
5.4	648

45.

SOLUTION:

x	y	$\frac{y}{x}$
0.2	24	120
0.6	72	120
1.8	216	120
5.4	648	120

Since $\frac{y}{x} = 120$ (a constant) for all x and y values in the relation, the relation is a direct variation.

x	y
12	18
24	36
36	18
72	9

46.

SOLUTION:

x	y	xy	$\dfrac{y}{x}$
12	18	216	1.5
24	36	864	1.5
36	18	648	0.5
72	9	648	0.125

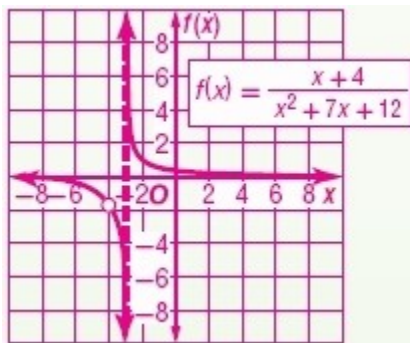Since neither xy or $\dfrac{y}{x}$ are constant, the relation is neither direct nor inverse variation.

Graph each function.

47. $f(x) = \dfrac{x+4}{x^2 + 7x + 12}$

SOLUTION:

$$f(x) = \dfrac{x+4}{x^2 + 7x + 12}$$
$$= \dfrac{x+4}{(x+4)(x+3)}$$
$$= \dfrac{1}{x+3}$$

The vertical asymptote is at $x = -3$ and there is a hole at $x = -4$. The horizontal asymptote is at $y = 0$.

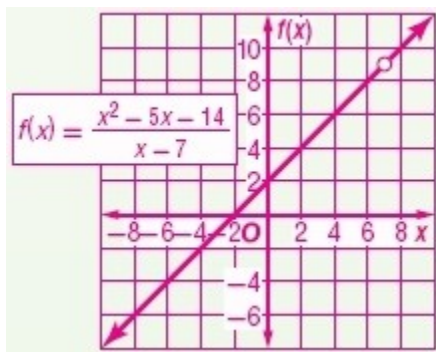48. $f(x) = \dfrac{x^2 - 5x - 14}{x - 7}$

SOLUTION:

$$f(x) = \dfrac{x^2 - 5x - 14}{x - 7}$$
$$= \dfrac{(x - 7)(x + 2)}{x - 7}$$
$$= x + 2$$

Therefore, the graph of $f(x) = \dfrac{x^2 - 5x - 14}{x - 7}$ is same as the graph of $f(x) = x + 2$ with a hole at $x = 7$.
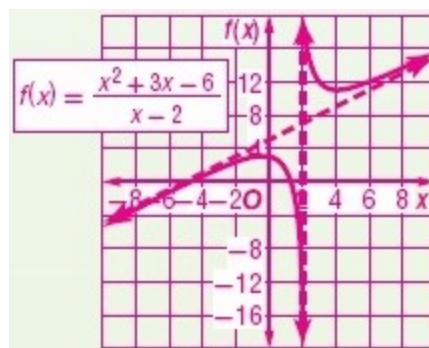

49. $f(x) = \dfrac{x^2 + 3x - 6}{x - 2}$

SOLUTION:

The vertical asymptote is at $x = 2$. Since the degree of the numerator is greater than the denominator, there is no horizontal asymptote.

$$\begin{array}{r} x + 5 \\ x - 2 \overline{)\, x^2 + 3x - 6} \\ \underline{x^2 - 2x } \\ 5x - 6 \\ \underline{5x - 10} \\ 4 \end{array}$$

Therefore, the oblique asymptote is $y = x + 5$.

50. **WEATHER** The atmospheric pressure P, in bars, of a given height on Earth is given by using the formula $P = a \cdot e^{-\frac{k}{H}}$. In the formula, a is the surface pressure on Earth, which is approximately 1 bar, k is the altitude for which you want to find the pressure in kilometers, and H is always 7 kilometers.

a. Find the pressure for 2, 4, and 7 kilometers.

b. What do you notice about the pressure as altitude increases?

SOLUTION:
a.

k	$P = e^{-\frac{k}{7}}$
2	$P = e^{-\frac{2}{7}}$ ≈ 0.75
4	$P = e^{-\frac{4}{7}}$ ≈ 0.56
7	$P = e^{-\frac{7}{7}}$ $= e^{-1}$ ≈ 0.37

b. The pressure decreases as the altitude increases.

51. **COMPUTERS** Since computers have been invented, computational speed has multiplied by a factor of 4 about every three years.

a. If a typical computer operates with a computational speed s today, write an expression for the speed at which you can expect an equivalent computer to operate after x three-year periods.

b. Suppose your computer operates with a processor speed of 2.8 gigahertz and you want a computer that can operate at 5.6 gigahertz. If a computer with that speed is currently unavailable for home use, how long can you expect to wait until you can buy such a computer?

SOLUTION:
a. $s \cdot 4^x$

b. Substitute 2.8 for s and 5.6 for $f(x)$ in the equation $f(x) = s \cdot 4^x$.

$$f(x) = s \cdot 4^x$$
$$5.6 = 2.8(4)^x$$
$$2 = 4^x$$
$$2 = 2^{2x}$$
$$1 = 2x$$
$$x = 0.5$$

So, we can expect 0.5 three-year periods or 1.5 year to buy 5.6 gigahertz speed home computers.

Determine whether the following are possible lengths of the sides of a right triangle.

52. 5, 12, 13

SOLUTION:

Because the longest side is 13 units, use 13 as c, the measure of the hypotenuse.

$$c^2 = a^2 + b^2$$
$$13^2 \overset{?}{=} 5^2 + 12^2$$
$$169 \overset{?}{=} 25 + 144$$
$$169 = 169 \checkmark$$

Because $c^2 = a^2 + b^2$, the triangle is a right triangle.

53. 60, 80, 100

SOLUTION:

Because the longest side is 100 units, use 100 as c, the measure of the hypotenuse.

$$c^2 = a^2 + b^2$$
$$100^2 \overset{?}{=} 60^2 + 80^2$$
$$10000 \overset{?}{=} 3600 + 6400$$
$$10000 = 10000 \checkmark$$

Because $c^2 = a^2 + b^2$, the triangle is a right triangle.

54. 7, 24, 25

SOLUTION:

Because the longest side is 25 units, use 25 as c, the measure of the hypotenuse.

$$c^2 = a^2 + b^2$$
$$25^2 \overset{?}{=} 24^2 + 7^2$$
$$625 \overset{?}{=} 576 + 49$$
$$625 = 625 \checkmark$$

Because $c^2 = a^2 + b^2$, the triangle is a right triangle.