

## 9-1 Midpoint and Distance Formulas

**CCSS PRECISION** Find the midpoint of the line segment with endpoints at the given coordinates.

1.  $(-4, 7), (3, 9)$

ANSWER:

$$\left(-\frac{1}{2}, 8\right)$$

2.  $(8, 2), (-1, -5)$

ANSWER:

$$(3.5, -1.5)$$

3.  $(11, 6), (18, 13.5)$

ANSWER:

$$(14.5, 9.75)$$

4.  $(-12, -2), (-10.5, -6)$

ANSWER:

$$(-11.25, -4)$$

Find the distance between each pair of points with the given coordinates.

5.  $(3, -5), (13, -11)$

ANSWER:

$$11.662 \text{ units}$$

6.  $(8, 1), (-2, 9)$

ANSWER:

$$12.806 \text{ units}$$

7.  $(0.25, 1.75), (3.5, 2.5)$

ANSWER:

$$3.335 \text{ units}$$

8.  $(-4.5, 10.75), (-6.25, -7)$

ANSWER:

$$17.836 \text{ units}$$

9. **MULTIPLE CHOICE** The map of a mall is overlaid with a numeric grid. The kiosk for the cell phone store is halfway between The Ice Creamery and the See Clearly eyeglass store. If the ice cream store is at  $(2, 4)$  and the eyeglass store is at  $(78, 46)$ , find the distance the kiosk is from the eyeglass store.

A 43.4 units

B 47.2 units

C 62.4 units

D 94.3 units

ANSWER:

A

Find the midpoint of the line segment with endpoints at the given coordinates.

10.  $(20, 3), (15, 5)$

ANSWER:

$$(17.5, 4)$$

11.  $(-27, 4), (19, -6)$

ANSWER:

$$(-4, -1)$$

12.  $(-0.4, 7), (11, -1.6)$

ANSWER:

$$(5.3, 2.7)$$

13.  $(5.4, -8), (9.2, 10)$

ANSWER:

$$(7.3, 1)$$

14.  $(-5.3, -8.6), (-18.7, 1)$

ANSWER:

$$(-12, -3.8)$$

15.  $(-6.4, -8.2), (-9.1, -0.8)$

ANSWER:

$$(-7.75, -4.5)$$

Find the distance between each pair of points with the given coordinates.

16.  $(1, 2), (6, 3)$

ANSWER:

$$5.099 \text{ units}$$

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17.  $(3, -4), (0, 12)$

ANSWER:

16.279 units

18.  $(-6, -7), (11, -12)$

ANSWER:

17.720 units

19.  $(-10, 8), (-8, -8)$

ANSWER:

16.125 units

20.  $(4, 0), (5, -6)$

ANSWER:

6.083 units

21.  $(7, 9), (-2, -10)$

ANSWER:

21.024 units

22.  $(-4, -5), (15, 17)$

ANSWER:

29.069 units

23.  $(14, -20), (-18, 25)$

ANSWER:

55.218 units

24. **TRACK AND FIELD** A shot put is thrown from the inside of a circle. A coordinate grid is placed over the shot put circle. The toe board is located at the front of the circle at  $(-4, 1)$ , and the back of the circle is at  $(5, 2)$ . If the center of the circle is halfway between these two points, what is the distance from the toe board to the center of the circle?

ANSWER:

$\sqrt{20.5} \approx 4.528$  units

**Find the midpoint of the line segment with endpoints at the given coordinates. Then find the distance between the points.**

25.  $(-93, 15), (90, -15)$

ANSWER:

$(-1.5, 0); 185.443$  units

26.  $(-22, 42), (57, 2)$

ANSWER:

$(17.5, 22); 88.549$  units

27.  $(-70, -87), (59, -14)$

ANSWER:

$(-5.5, -50.5); 148.223$  units

28.  $(-98, 5), (-77, 64)$

ANSWER:

$(-87.5, 34.5); 62.626$  units

29.  $(41, -45), (-25, 75)$

ANSWER:

$(8, 15); 136.953$  units

30.  $(90, 60), (-3, -2)$

ANSWER:

$(43.5, 29); 111.772$  units

31.  $(-1.2, 2.5), (0.34, -7)$

ANSWER:

$(-0.43, -2.25); 9.624$  units

32.  $(-7.54, 3.89), (4.04, -0.38)$

ANSWER:

$(-1.75, 1.755); 12.342$  units

33.  $\left(-\frac{5}{12}, -\frac{1}{3}\right), \left(-\frac{17}{2}, -\frac{5}{3}\right)$

ANSWER:

$(-4.458, -1); 8.193$  units

34.  $\left(-\frac{5}{4}, -\frac{13}{2}\right), \left(-\frac{4}{3}, -\frac{5}{6}\right)$

ANSWER:

$(-1.292, -3.667); 5.667$  units

35.  $(-3\sqrt{2}, -4\sqrt{5}), (-3\sqrt{3}, 9)$

ANSWER:

$(-4.719, 0.028); 17.97$  units

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36.  $\left(\frac{\sqrt{3}}{3}, \frac{\sqrt{2}}{4}\right), \left(\frac{-2\sqrt{3}}{3}, \frac{\sqrt{2}}{4}\right)$

ANSWER:

$(-0.289, 0.354); \sqrt{3}$  units

37. **SPACE** Use the labeled points on the outline of the circular crater on Mars to estimate its diameter in kilometers. Assume each unit on the coordinate system is 1 kilometer.

Refer to the photo on Page 596.

ANSWER:

14.53 km

38. **CCSS MODELING** Triangle  $ABC$  has vertices  $A(2, 1)$ ,  $B(-6, 5)$ , and  $C(-2, -3)$ .

- An isosceles triangle has two sides with equal length. Is triangle  $ABC$  isosceles? Explain.
- An equilateral triangle has three sides of equal length. Is triangle  $ABC$  equilateral? Explain.
- Triangle  $EFG$  is formed by joining the midpoints of the sides of triangle  $ABC$ . What type of triangle is  $EFG$ ? Explain.
- Describe any relationship between the lengths of the sides of the two triangles.

ANSWER:

a. Yes;  $AB = BC = 4\sqrt{5}$ .

b. No;  $AC = 4\sqrt{2}$ .

c. Triangle  $EFG$  is also isosceles with two sides of measure  $2\sqrt{5}$ .

d. The side lengths of  $\triangle EFG$  are one half the side lengths of the sides of  $\triangle ABC$ .

39. **PACKAGE DELIVERY**

To determine the mileage between cities for their overnight delivery service, a package delivery service superimposes a coordinate grid over the United States. Each side of a grid unit is equal to 0.316 mile. Suppose the locations of two distribution centers are at  $(132, 428)$  and  $(254, 105)$ . Find the actual distance between these locations to the nearest mile.

ANSWER:

109 mi

40. **HIKING** Orlando wants to hike from his camp to a waterfall. The waterfall is 5 miles south and 8 miles east of his campsite.

- Use the Distance Formula to determine how far the waterfall is from the campsite.
- Verify your answer in part a by using the Pythagorean Theorem to determine the distance between the campsite and the waterfall.
- Orlando wants to stop for lunch halfway to the waterfall. If the camp is at the origin, where should he stop?

ANSWER:

a.  $\sqrt{89} \approx 9.4$  mi

b.

$$8^2 + 5^2 = c^2$$

$$64 + 25 = c^2$$

$$89 = c^2$$

$$c = \sqrt{89}$$

or about 9.4 mi

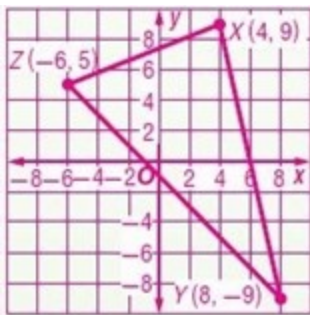
c.  $\left(4, -\frac{5}{2}\right)$

## 9-1 Midpoint and Distance Formulas

41. **MULTIPLE REPRESENTATIONS** Triangle  $XYZ$  has vertices  $X(4, 9)$ ,  $Y(8, -9)$ , and  $Z(-6, 5)$ .
- CONCRETE** Draw  $\triangle XYZ$  on a coordinate plane.
  - NUMERICAL** Find the coordinates of the midpoint of each side of the triangle.
  - GEOMETRIC** Find the perimeter of  $\triangle XYZ$  and the perimeter of the triangle with vertices at the points found in part **b**.
  - ANALYTICAL** How do the perimeters in part **c** compare?

ANSWER:

a.



- midpoint of  $\overline{XY} = (6, 0)$ ;  
midpoint of  $\overline{YZ} = (1, -2)$ ;  
midpoint of  $\overline{XZ} = (-1, 7)$
  - The perimeter of  $\triangle XYZ$  is  $2\sqrt{29} + 14\sqrt{2} + 2\sqrt{85}$  units.  
perimeter =  $\sqrt{29} + 7\sqrt{2} + \sqrt{85}$ .
  - The perimeter of  $\triangle XYZ$  is twice the perimeter of the smaller triangle.
42. **CHALLENGE** Find the coordinates of the point that is three fourths of the way from  $P(-1, 12)$  to  $Q(5, -10)$ .

ANSWER:

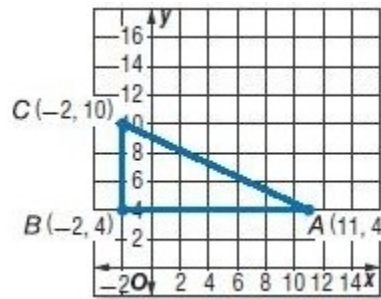
$$\left(\frac{7}{2}, -\frac{9}{2}\right)$$

43. **REASONING** Identify all the points in a plane that are three units or less from the point  $(5, 6)$ . What figure does this make?

ANSWER:

a circle and its interior with center at  $(5, 6)$  and radius 3 units

44. **CCSS ARGUMENTS** Triangle  $ABC$  is a right triangle.
- Find the midpoint of the hypotenuse. Call it point  $Q$ .
  - Classify  $\triangle BQC$  according to the lengths of its sides. Include sufficient evidence to support your conclusion.
  - Classify  $\triangle BQA$  according to its angles.



ANSWER:

a.  $Q\left(\frac{9}{2}, 7\right)$

- $BC = 6$ ,  $QC = \frac{\sqrt{205}}{2}$ ,  $BQ = \frac{\sqrt{205}}{2}$ ;  $\overline{QC}$  and  $\overline{BQ}$  are the same length; therefore,  $\triangle BQC$  is isosceles.
- Obtuse

45. **OPEN ENDED** Plot two points, and find the distance between them. Does it matter which ordered pair is first when using the Distance Formula? Explain.

ANSWER:

See students' graphs; the distance from  $A$  to  $B$  equals the distance from  $B$  to  $A$ . Using the Distance Formula, the solution is the same no matter which ordered pair is used first.

46. **WRITING IN MATH** Explain how the Midpoint Formula can be used to approximate the halfway point between two locations on a map.

ANSWER:

Most maps have a superimposed grid. Think of the grid as a coordinate system and assign approximate coordinates to the two locations. Then use the Midpoint Formula to find the midpoint between the points with those coordinates.

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47. **SHORT RESPONSE** You currently earn \$8.10 per hour and your boss gives you a 10% raise. What is your new hourly wage?

**ANSWER:**

\$8.91

48. **SAT/ACT** A right circular cylinder has a radius of 3 and a height of 5. Which of the following dimensions of a rectangular solid will have a volume closest to that of the cylinder?

**A** 5, 5, 6

**B** 5, 6, 6

**C** 5, 5, 5

**D** 4, 5, 6

**E** 3, 5, 9

**ANSWER:**

E

49. **GEOMETRY** If the sum of the lengths of the two legs of a right triangle is 49 inches and the hypotenuse is 41 inches, find the longer of the two legs.

**F** 9 in.

**G** 40 in.

**H** 42 in.

**J** 49 in.

**ANSWER:**

G

50. Five more than 3 times a number is 17. Find the number.

**A** 3

**B** 4

**C** 5

**D** 6

**ANSWER:**

B

**Solve each equation. Check your solutions.**

51.  $\frac{12}{v^2 - 16} - \frac{24}{v - 4} = 3$

**ANSWER:**

-6, -2

52.  $\frac{w}{w-1} + w = \frac{4w-3}{w-1}$

**ANSWER:**

3

53.  $\frac{4n^2}{n^2 - 9} - \frac{2n}{n + 3} = \frac{3}{n - 3}$

**ANSWER:**

$\frac{3}{2}$

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54. **SWIMMING** When a person swims underwater, the pressure in his or her ears varies directly with the depth at which he or she is swimming.

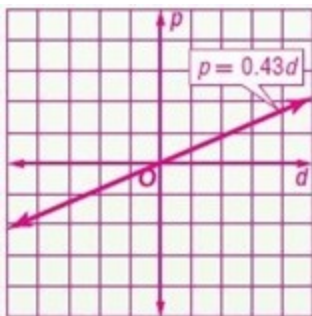


- Write a direct variation equation that represents this situation.
- Find the pressure at 60 feet.
- It is unsafe for amateur divers to swim where the water pressure is more than 65 pounds per square inch. How deep can an amateur diver safely swim?
- Make a table showing the number of pounds of pressure at various depths of water. Use the data to draw a graph of pressure versus depth.

**ANSWER:**

- $P = 0.43d$
- 25.8 psi
- about 151 ft
- 

Depth(ft)	
0	0
1	0.43
2	0.86
3	1.29
4	1.72



**Solve each equation or inequality. Round to the nearest ten-thousandth.**

55.  $9^{z-4} = 6.28$

**ANSWER:**

4.8362

56.  $8.2^{n-3} = 42.5$

**ANSWER:**

4.7820

57.  $2.1^{t-5} = 9.32$

**ANSWER:**

8.0086

58.  $8^{2n} > 52^{4n+3}$

**ANSWER:**

$\{n | n > -1.0178\}$

59.  $7^{p+2} \leq 13^{5-p}$

**ANSWER:**

$\{p | p \leq 1.9803\}$

60.  $3^{y+2} \geq 8^{3y}$

**ANSWER:**

$\{y | y \leq 0.4275\}$

**Solve each equation.**

61.  $(6n - 5)^{\frac{1}{3}} + 3 = -2$

**ANSWER:**

-20

62.  $(5x + 7)^{\frac{1}{5}} + 3 = 5$

**ANSWER:**

5

63.  $(3x - 2)^{\frac{1}{5}} + 6 = 5$

**ANSWER:**

$\frac{1}{3}$

## 9-1 Midpoint and Distance Formulas

**Write each quadratic equation in vertex form.  
Then identify the vertex, axis of symmetry, and  
direction of opening.**

64.  $y = -x^2 - 4x + 8$

*ANSWER:*

$$y = -(x + 2)^2 + 12; (-2, 12); x = -2; \text{down}$$

65.  $y = x^2 - 6x + 1$

*ANSWER:*

$$y = (x - 3)^2 - 8; (3, -8); x = 3; \text{up}$$

66.  $y = -2x^2 + 20x - 35$

*ANSWER:*

$$y = -2(x - 5)^2 + 15; (5, 15); x = 5; \text{down}$$