9-2 Parabolas

Write each equation in standard form. Identify the vertex, axis of symmetry, and direction of opening of the parabola.

1. $y = 2x^2 - 24x + 40$

ANSWER:

$y = 2(x - 6)^2 - 32$; vertex (6, –32); axis of symmetry: $x = 6$; opens upward

2. $y = 3x^2 - 6x - 4$

ANSWER:

$y = 3(x - 1)^2 - 7$; vertex (1, –7); axis of symmetry: $x = 1$; opens upward

3. $x = y^2 - 8y - 11$

ANSWER:

$x = (y - 4)^2 - 27$; vertex (–27, 4); axis of symmetry: $y = 4$; opens right

4. $x + 3y^2 + 12y = 18$

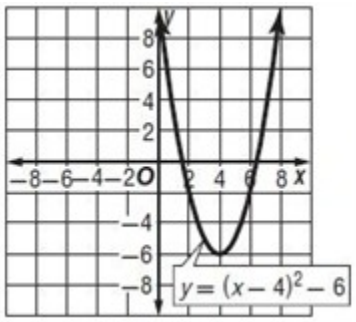
ANSWER:

$x = -3(y + 2)^2 + 30$; vertex (30, –2); axis of symmetry: $y = -2$; opens left

Graph each equation.

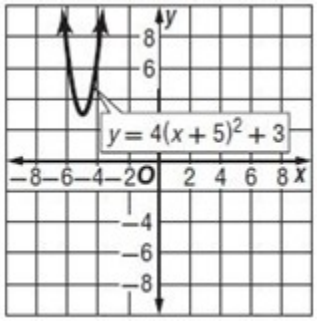
5. $y = (x - 4)^2 - 6$

ANSWER:



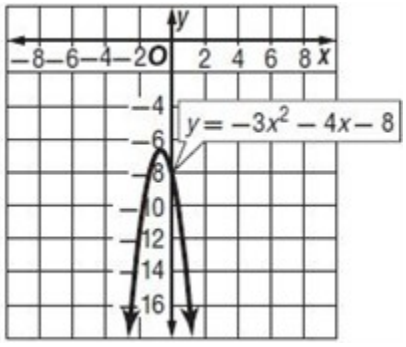
6. $y = 4(x + 5)^2 + 3$

ANSWER:



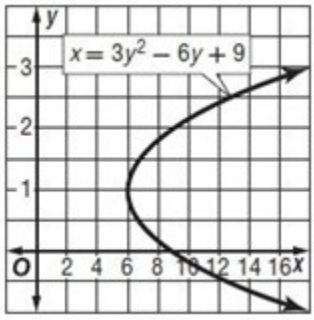
7. $y = -3x^2 - 4x - 8$

ANSWER:



8. $x = 3y^2 - 6y + 9$

ANSWER:

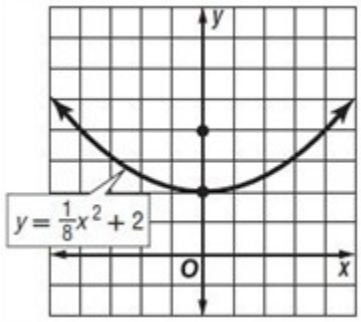


Write an equation for each parabola described below. Then graph the equation.

9. vertex (0, 2), focus (0, 4)

ANSWER:

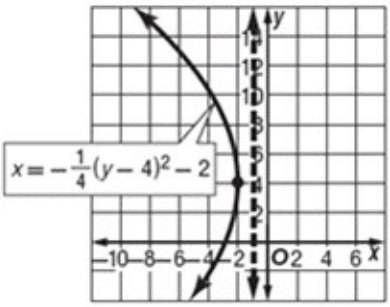
$y = \frac{1}{8}x^2 + 2$



10. vertex (–2, 4), directrix $x = –1$

ANSWER:

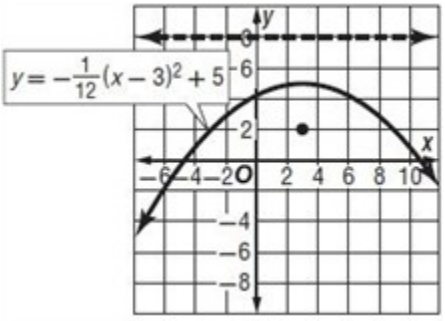
$x = -\frac{1}{4}(y-4)^2 - 2$



11. focus (3, 2), directrix $y = 8$

ANSWER:

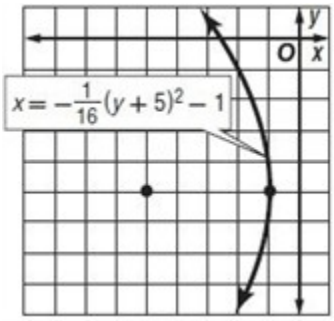
$y = -\frac{1}{12}(x-3)^2 + 5$



12. vertex (–1, –5), focus (–5, –5)

ANSWER:

$x = -\frac{1}{16}(y+5)^2 - 1$

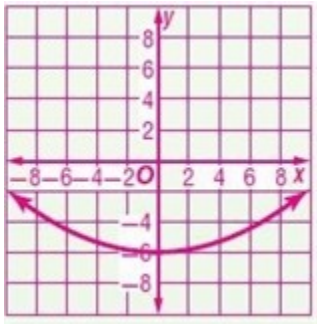


13. **ASTRONOMY** Consider a parabolic mercury mirror like the one described at the beginning of the lesson. The focus is 6 feet above the vertex and the latus rectum is 24 feet long.
a. Assume that the focus is at the origin. Write an equation for the parabola formed by the parabolic microphone.
b. Graph the equation

ANSWER:

a. $y = \frac{1}{24}x^2 - 6$

b.



Write each equation in standard form. Identify the vertex, axis of symmetry, and direction of opening of the parabola.

14. $y = x^2 - 8x + 13$

ANSWER:

$y = (x - 4)^2 - 3$; vertex = (4, –3); axis of symmetry: $x = 4$; opens upward

15. $y = 3x^2 + 42x + 149$

ANSWER:

$y = 3(x + 7)^2 + 2$; vertex = (–7, 2); axis of symmetry: x = –7; opens upward

16. $y = -6x^2 - 36x - 8$

ANSWER:

$y = -6(x + 3)^2 + 46$; vertex = (–3, 46); axis of symmetry: x = –3; opens downward

17. $y = -3x^2 - 9x - 6$

ANSWER:

$y = -3\left(x + \frac{3}{2}\right)^2 + \frac{3}{4}$; vertex = $\left(-\frac{3}{2}, \frac{3}{4}\right)$; axis of symmetry: $x = -\frac{3}{2}$; opens downward

18. $x = \frac{1}{3}y^2 - 3y + 4$

ANSWER:

$x = \frac{1}{3}(y - 4.5)^2 - 2.75$; vertex = $(-2.75, 4.5)$; axis of symmetry: y = 4.5; opens right

19. $x = \frac{2}{3}y^2 - 4y + 12$

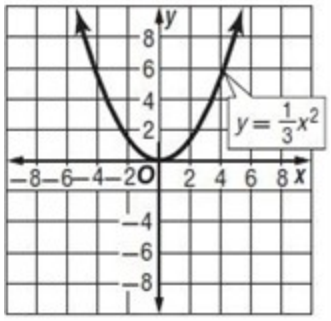
ANSWER:

$x = \frac{2}{3}(y - 3)^2 + 6$; vertex = (6, 3); axis of symmetry: y = 3; opens right

Graph each equation.

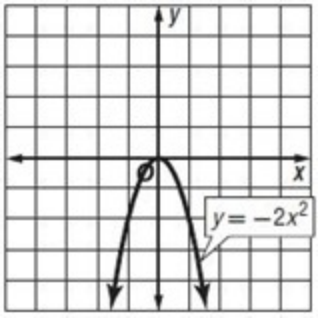
20. $y = \frac{1}{3}x^2$

ANSWER:



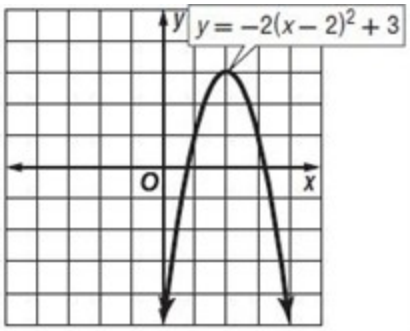
21. $y = -2x^2$

ANSWER:



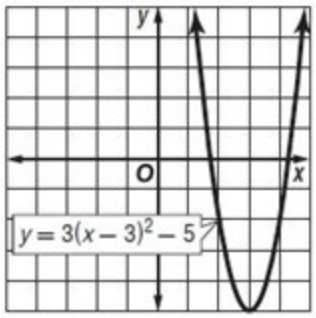
22. $y = -2(x - 2)^2 + 3$

ANSWER:



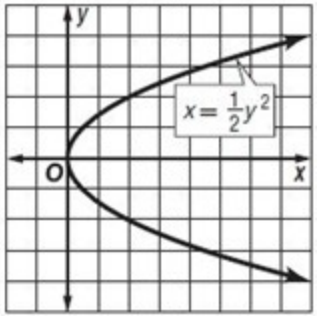
23. $y = 3(x-3)^2 - 5$

ANSWER:



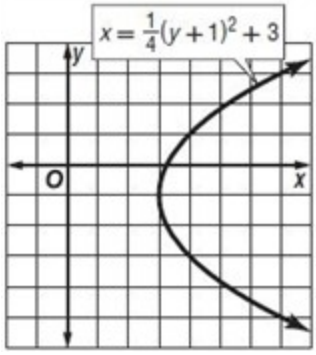
24. $x = \frac{1}{2}y^2$

ANSWER:



25. $4x - y^2 = 2y + 13$

ANSWER:

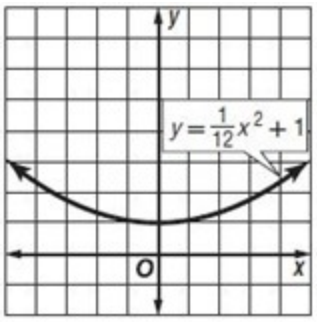


Write an equation for each parabola described below. Then graph the equation.

26. vertex (0, 1), focus (0, 4)

ANSWER:

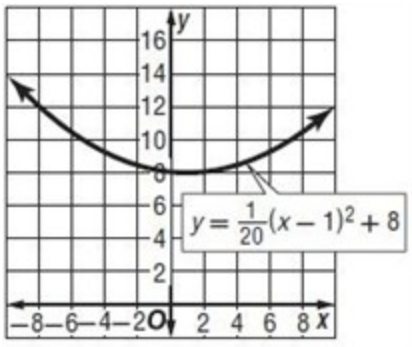
$y = \frac{1}{12}x^2 + 1$



27. vertex (1, 8), directrix $y = 3$

ANSWER:

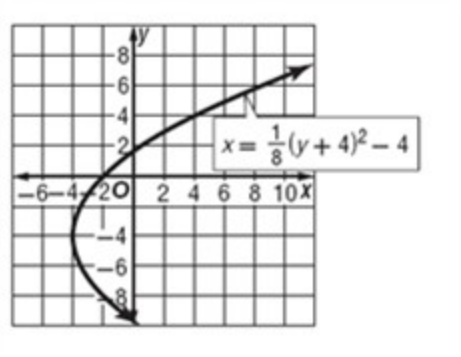
$y = \frac{1}{20}(x-1)^2 + 8$



28. focus (–2, –4), directrix $x = –6$

ANSWER:

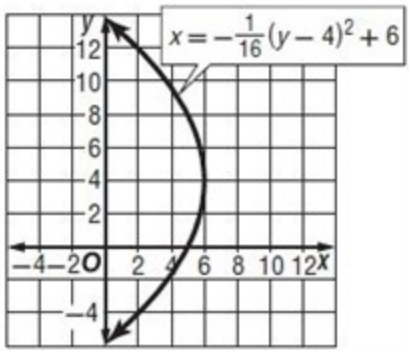
$$x = \frac{1}{8}(y+4)^2 - 4$$



29. focus (2, 4), directrix $x = 10$

ANSWER:

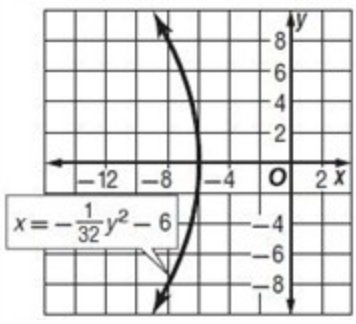
$$x = -\frac{1}{16}(y-4)^2 + 6$$



30. vertex (–6, 0), directrix $x = 2$

ANSWER:

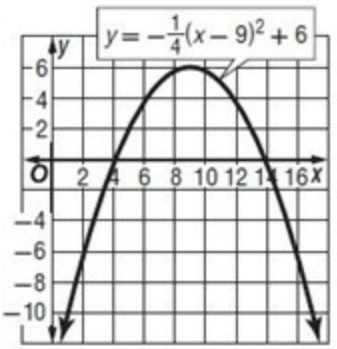
$$x = -\frac{1}{32}y^2 - 6$$



31. vertex (9, 6), focus (9, 5)

ANSWER:

$$y = -\frac{1}{4}(x-9)^2 + 6$$



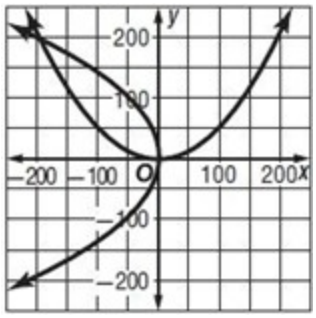
32. **BASEBALL** When a ball is thrown, the path it travels is a parabola. Suppose a baseball is thrown from ground level, reaches a maximum height of 50 feet, and hits the ground 200 feet from where it was thrown. Assuming this situation could be modeled on a coordinate plane with the focus of the parabola at the origin, find the equation of the parabolic path of the ball. Assume the focus is on ground level.

ANSWER:

$$y = -\frac{1}{200}x^2 + 50$$

33. **CCSS PERSEVERANCE** Ground antennas and satellites are used to relay signals between the NASA Mission Operations Center and the spacecraft it controls. One such parabolic dish is 146 feet in diameter. Its focus is 48 feet from the vertex.
a. Sketch two options for the dish, one that opens up and one that opens left.
b. Write two equations that model the sketches in part **a**.
c. If you wanted to find the depth of the dish, does it matter which equation you use? Why or why not?

ANSWER:
a.



b. $y = \frac{x^2}{192}$ and $x = \frac{y^2}{-192}$
c. Sample answer: No; except for the direction in which they open, the graphs are identical.

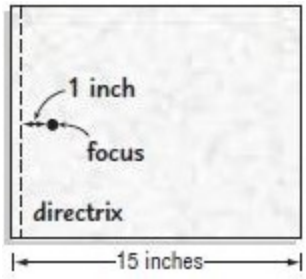
34. **UMBRELLAS** A beach umbrella has an arch in the shape of a parabola that opens downward. The umbrella spans 6 feet across and is $1\frac{1}{2}$ feet high. Write an equation of a parabola to model the arch, assuming that the origin is at the point where the pole and umbrella meet, beneath the vertex of the arch.

ANSWER:
$y = -\frac{1}{6}x^2 + \frac{3}{2}$

35. **AUTOMOBILES** An automobile headlight contains a parabolic reflector. The light coming from the source bounces off the parabolic reflector and shines out the front of the headlight. The equation of the cross section of the reflector is $y = \frac{1}{12}x^2$. How far from the vertex should the filament for the high beams be placed?

ANSWER:
3 units

36. **MULTIPLE REPRESENTATIONS** Start with a sheet of wax paper that is about 15 inches long and 12 inches wide.
a. CONCRETE Make a line that is perpendicular to the sides of the sheet by folding the sheet near one end. Open up the paper again. This line is the directrix. Mark a point about midway between the sides of the sheet so that the distance from the directrix is about 1 inch. This is the focus.
b. CONCRETE Start with a new sheet of wax paper. Form another outline of a parabola with a focus that is about 3 inches from the directrix.
c. CONCRETE On a new sheet of a wax paper, form a third outline of a parabola with a focus that is about 5 inches from the directrix.
d. VERBAL Compare the shapes of the three parabolas. How does the distance between the focus and the directrix affect the shape of a parabola?



ANSWER:
a–c. See students' work.
d. As the distance between the directrix and the focus increases, the parabola becomes wider.

37. **REASONING** How do you change the equation of the parent function $y = x^2$ to shift the graph to the right

ANSWER:

Rewrite it as $y = (x - h)^2$, where h > 0.

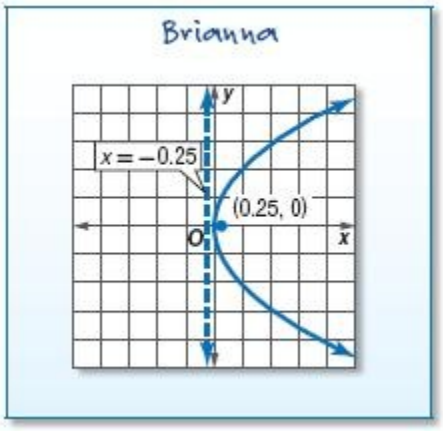
38. **OPEN ENDED** Two different parabolas have their vertex at (–3, 1) and contain the point with coordinates (–1, 0). Write two possible equations for these parabolas.

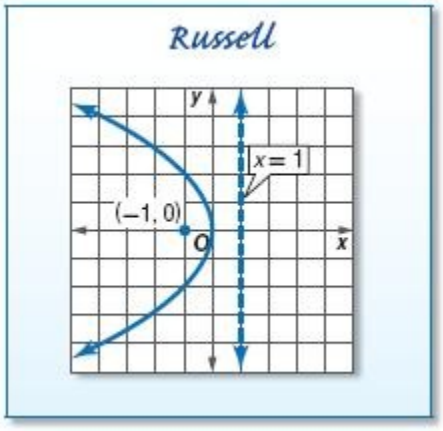
ANSWER:

Sample answers:

$y = -\frac{1}{4}(x+3)^2 + 1$ and $x = 2(y-1)^2 - 3$

39. **CCSS CRITIQUE** Brianna and Russell are graphing $\frac{1}{4}y^2 + x = 0$. Is either of them correct? Explain your reasoning.





ANSWER:

Russell; the parabola should open to the left rather than to the right.

40. **WRITING IN MATH** Why are parabolic shapes used in the real world?

ANSWER:

Sample answer: When rays which are parallel to the axis of symmetry of a parabolic mirror are reflected off the mirror, they are directed to the focus. This focuses all of the rays at one specific point. Also, a parabolic microphone can be used to make capturing sound more effective, because reflected sound waves are focused at the particular point where the microphone is located.

41. A gardener is placing a fence around a 1320-square-foot rectangular garden. He ordered 148 feet of fencing. If he uses all the fencing, what is the length of the longer side of the garden?

A 30 ft
B 34 ft
C 44 ft
D 46 ft

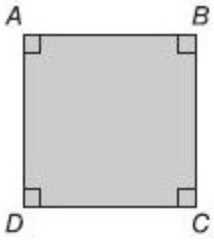
ANSWER:
C

42. **SAT/ACT** When a number is divided by 5, the result is 7 more than the number. Find the number.

F $-\frac{35}{4}$
G $-\frac{35}{6}$
H $\frac{35}{6}$
J $\frac{28}{4}$
K $\frac{35}{4}$

ANSWER:
F

43. **GEOMETRY** What is the area of the following square, if the length of $\overline{BD}$ is $2\sqrt{2}$?



A 1
B 2
C 3
D 4

ANSWER:
D

44. **SHORT RESPONSE** The measure of the smallest angle of a triangle is two thirds the measure of the middle angle. The measure of the middle angle is three sevenths of the measure of the largest angle. Find the largest angle's measure.

ANSWER:
105°

45. **GEOMETRY** Find the perimeter of a triangle with vertices at (2, 4), (–1, 3), and (1, –3).

ANSWER:
$5\sqrt{2}+3\sqrt{10}$ units

46. **WORK** A worker can powerwash a wall of a certain size in 5 hours. Another worker can do the same job in 4 hours. If the workers work together, how long would it take to do the job? Determine whether your answer is reasonable.

ANSWER:
$2\frac{2}{9}$ h ; The answer is reasonable. The time to complete the job when working together must be less than the time it would take either person working alone.

Solve each equation or inequality. Round to the nearest ten-thousandth.

47. $\ln(x + 1) = 1$

ANSWER:
1.7183

48. $\ln(x - 7) = 2$

ANSWER:
14.3891

49. $e^x > 1.6$

ANSWER:
$x > 0.4700$

50. $e^{5x} \geq 25$

ANSWER:
$x \geq 0.6438$

Simplify.

51. $\sqrt{0.25}$

ANSWER:
0.5

52. $\sqrt[3]{-0.064}$

ANSWER:
–0.4

53. $\sqrt[4]{z^8}$

ANSWER:
z^2

54. $-\sqrt[6]{x^6}$

ANSWER:
$-|x|$

List all of the possible rational zeros of each function.

55. $h(x) = x^3 + 8x + 6$

ANSWER:
$\pm 1, \pm 2, \pm 3, \pm 6$

56. $p(x) = 3x^3 - 5x^2 - 11x + 3$

ANSWER:
$\pm 1, \pm \frac{1}{3}, \pm 3$

57. $h(x) = 9x^6 - 5x^3 + 27$

ANSWER:
$\pm 1, \pm \frac{1}{3}, \pm \frac{1}{9}, \pm 3, \pm 9, \pm 27$

Simplify each expression.

58. $\sqrt{24}$

ANSWER:
$2\sqrt{6}$

59. $\sqrt{45}$

ANSWER:
$3\sqrt{5}$

60. $\sqrt{252}$

ANSWER:
$6\sqrt{7}$

61. $\sqrt{512}$

ANSWER:
$16\sqrt{2}$