9-3 Circles

1. **WEATHER** On average, the eye of a tornado is about 200 feet across. Suppose the center of the eye is at the point (72, 39). Write an equation to represent the boundary of the eye.

   ANSWER:

   $(x-72)^2+(y-39)^2=10{,}000$

Write an equation for each circle given the center and radius.

2. center: (–2, –6), r = 4 units

   ANSWER:

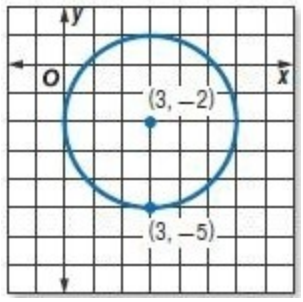
   $(x+2)^2+(y+6)^2=16.$

3. center: (1,–5), r = 3 units

   ANSWER:

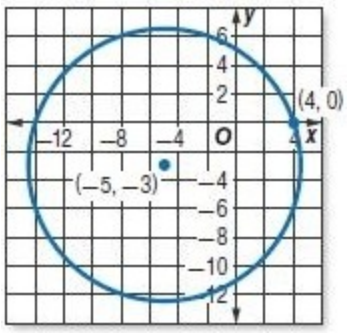
   $(x-1)^2+(y+5)^2=9.$

Write an equation for each graph.

4. 

   ANSWER:

   $(x-3)^2+(y+2)^2=9$

5. 

   ANSWER:

   $(x+5)^2+(y+3)^2=90$

Write an equation for each circle given the endpoints of a diameter.

6. (–1, –7) and (0, 0)

   ANSWER:

   $\left(x+\frac{1}{2}\right)^2+\left(y+\frac{7}{2}\right)^2=\frac{25}{2}$

7. (4, –2) and (–4, –6)

   ANSWER:

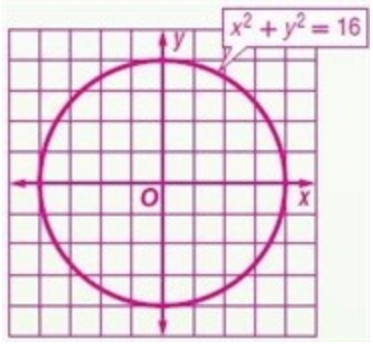
   $x^2+(y+4)^2=20$

Find the center and radius of each circle. Then graph the circle.

8. $x^2+y^2=16$

   ANSWER:

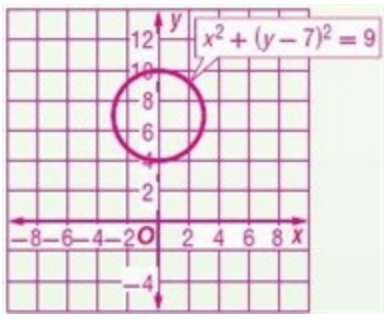
   center: (0, 0); radius: 4

9. $x^2+(y-7)^2=9$

   ANSWER:

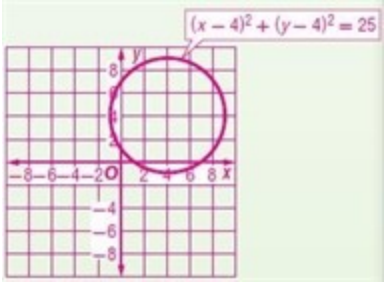
   center: (0, 7); radius: 3

10. $(x-4)^2+(y-4)^2=25$

ANSWER:

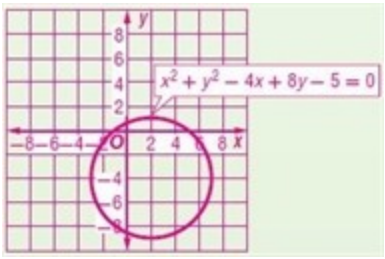
center: (4, 4); radius: 5



11. $x^2+y^2-4x+8y-5=0$

ANSWER:

center: (2, –4); radius: 5



Write an equation for each circle given the center and radius.

12. center: (4, 9), $r = 6$

ANSWER:

$(x-4)^2+(y-9)^2=36.$

13. center: (–3, 1), $r = 4$

ANSWER:

$(x+3)^2+(y-1)^2=16$

14. center: (–7, –3), $r = 13$

ANSWER:

$(x+7)^2+(y+3)^2=169$

15. center: (–2, –1), $r = 9$

ANSWER:

$(x+2)^2+(y+1)^2=81$

16. center: (1, 0), $r = \sqrt{15}$

ANSWER:

$(x-1)^2+y^2=15$

17. center: (0, –6), $r = \sqrt{35}$

ANSWER:

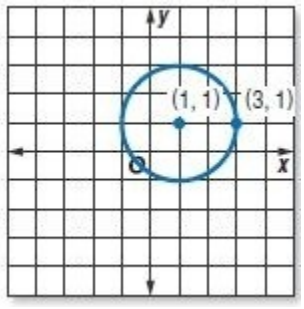
$x^2+(y+6)^2=35$

18. **CCSS MODELING** The radar for a county airport control tower is located at (5, 10) on a map. It can detect a plane up to 20 miles away. Write an equation for the outer limits of the detection area.

ANSWER:

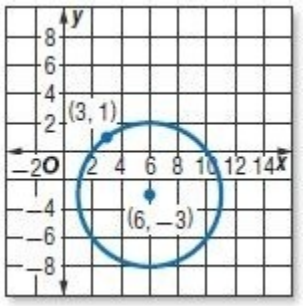
$(x-5)^2+(y-10)^2=400$

Write an equation for each graph.

19. 

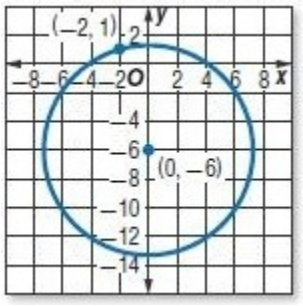
ANSWER:

$(x-1)^2+(y-1)^2=4$

20. 

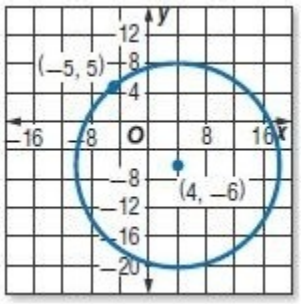
ANSWER:

$(x-6)^2+(y+3)^2=25$

21. 

ANSWER:

$x^2 + (y+6)^2 = 53$

22. 

ANSWER:

$(x-4)^2 + (y+6)^2 = 202$

Write an equation for each circle given the endpoints of a diameter.

23. (2, 1) and (2, –4)

ANSWER:

$(x-2)^2 + \left(y+\frac{3}{2}\right)^2 = \frac{25}{4}$

24. (–4, –10) and (4, –10)

ANSWER:

$x^2 + (y+10)^2 = 16$

25. (5, –7) and (–2, –9)

ANSWER:

$\left(x-\frac{3}{2}\right)^2 + (y+8)^2 = \frac{53}{4}$

26. (–6, 4) and (4, 8)

ANSWER:

$(x+1)^2 + (y-6)^2 = 29$

27. (2, –5) and (6, 3)

ANSWER:

$(x-4)^2 + (y+1)^2 = 20$

28. (18, 11) and (–19, –13)

ANSWER:

$\left(x+\frac{1}{2}\right)^2 + (y+1)^2 = 486\frac{1}{4}$

29. **LAWN CARE** A sprinkler waters a circular section of lawn.
a. Write an equation to represent the boundary of the sprinkler area if the endpoints of a diameter are at (–12, 16) and (12, –16).
b. What is the area of the lawn that the sprinkler waters?

ANSWER:

a. $x^2 + y^2 = 400$

b. approximately 1256.64 units^2.

30. **SPACE** Apollo 8 was the first manned spacecraft to orbit the Moon at an average altitude of 185 kilometers above the Moon's surface. Write an equation to model a single circular orbit of the command module if the endpoints of a diameter are at (1740, 0) of the Moon and (–1740, 0). Let the center of the Moon be at the origin of the coordinate plane measured in kilometers.

ANSWER:

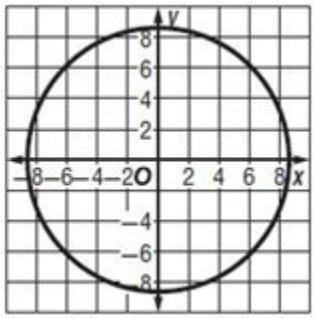
$x^2 + y^2 = 3,705,625$

Find the center and radius of each circle. Then graph the circle.

31. $x^2 + y^2 = 75$

ANSWER:

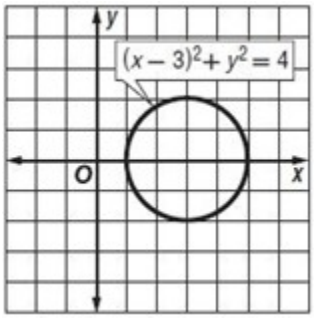
center: (0,0); radius: $5\sqrt{3}$



32. $(x-3)^2 + y^2 = 4$

ANSWER:

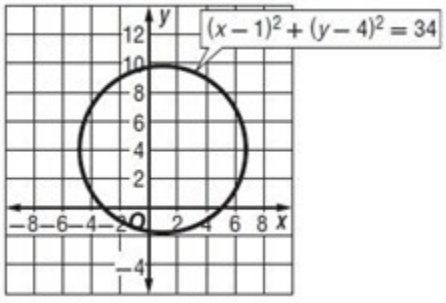
center: (3, 0); radius: 2



33. $(x-1)^2 + (y-4)^2 = 34$

ANSWER:

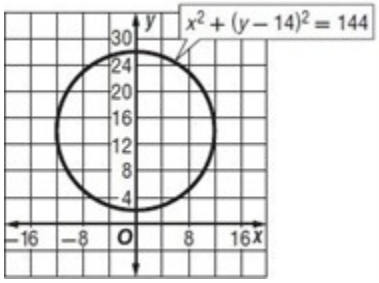
center: (1, 4); radius: $\sqrt{34}$



34. $x^2 + (y-14)^2 = 144$

ANSWER:

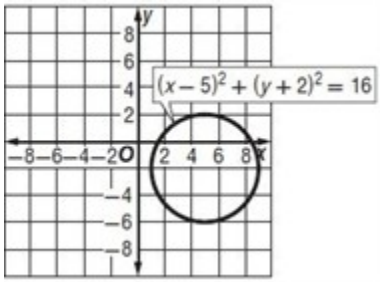
center: (0, 14); radius: 12



35. $(x-5)^2 + (y+2)^2 = 16$

ANSWER:

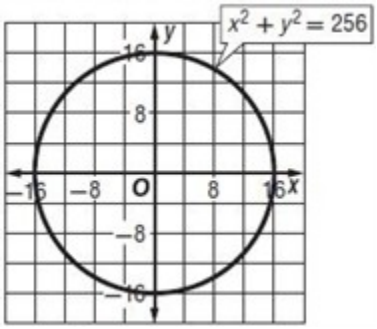
center: (5, –2); radius: 4



36. $x^2 + y^2 = 256$

ANSWER:

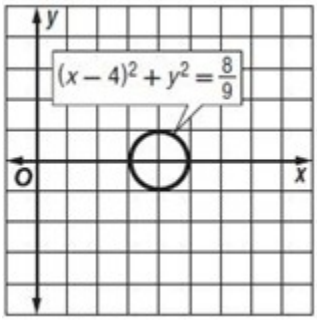
center: (0, 0); radius: 16



37. $(x-4)^2+y^2=\frac{8}{9}$

ANSWER:

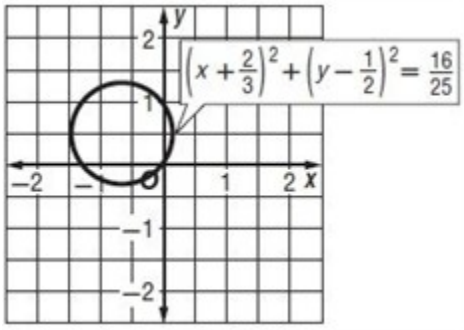
center: (4, 0); radius: $\frac{\sqrt{8}}{3}$



38. $\left(x+\frac{2}{3}\right)^2+\left(y-\frac{1}{2}\right)^2=\frac{16}{25}$

ANSWER:

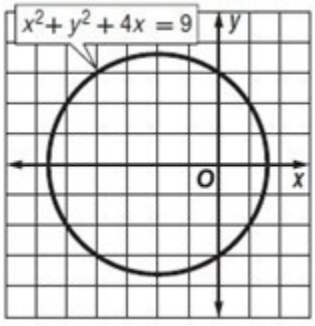
center: $\left(-\frac{2}{3},\frac{1}{2}\right)$; radius: $\frac{4}{5}$



39. $x^2+y^2+4x=9$

ANSWER:

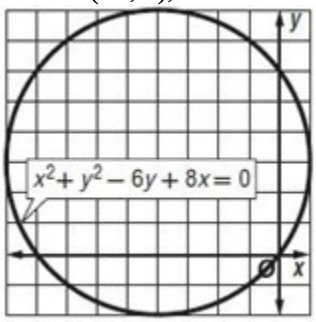
center: (–2, 0); radius: $\sqrt{13}$



40. $x^2+y^2-6y+8x=0$

ANSWER:

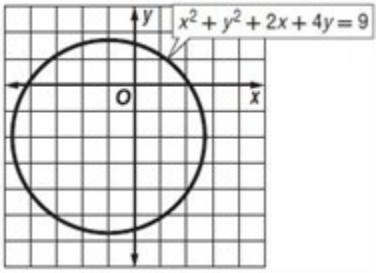
center: (–4, 3); radius: 5



41. $x^2+y^2+2x+4y=9$

ANSWER:

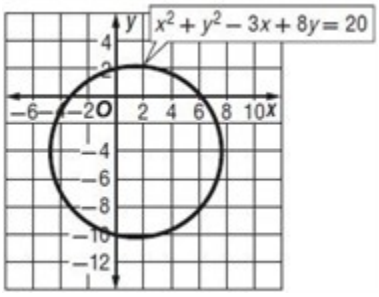
center: (–1, –2); radius: $\sqrt{14}$



42. $x^2+y^2-3x+8y=20$

ANSWER:

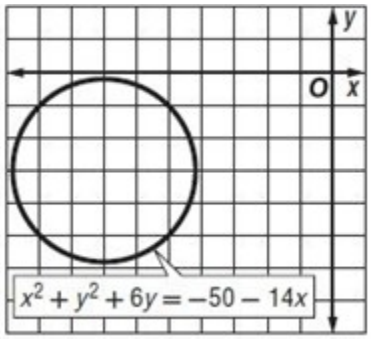
center: $\left(\frac{3}{2},-4\right)$; radius: $\frac{3\sqrt{17}}{2}$



43. $x^2 + y^2 + 6y = -50 - 14x$

ANSWER:

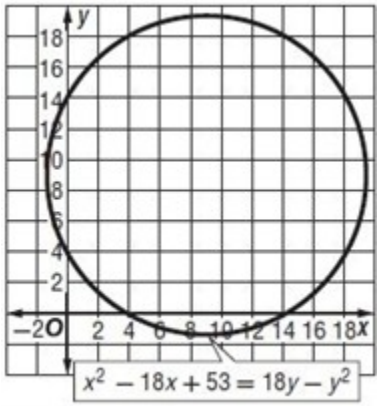
center: (–7, –3); radius: $2\sqrt{2}$ units



44. $x^2 - 18x + 53 = 18y - y^2$

ANSWER:

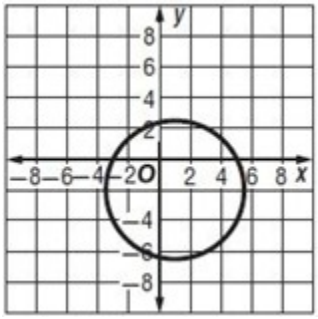
center: (9, 9); radius: $\sqrt{109}$ units



45. $2x^2 + 2y^2 - 4x + 8y = 32$

ANSWER:

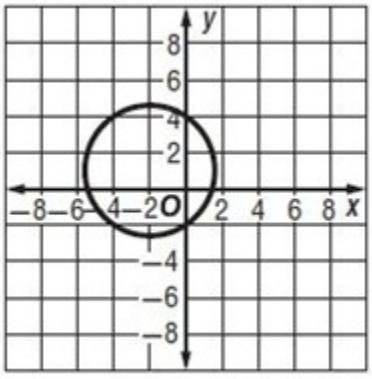
center: (1, –2); radius: $\sqrt{21}$



46. $3x^2 + 3y^2 - 6y + 12x = 24$

ANSWER:

center: (–2, 1); radius: $\sqrt{13}$

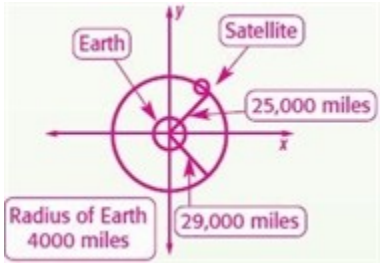


47. **SPACE** A satellite is in a circular orbit 25,000 miles above Earth.
a. Write an equation for the orbit of this satellite if the origin is at the center of Earth.
Use 8000 miles as the diameter of Earth.
b. Draw a sketch of Earth and the orbit to scale. Label your sketch.

ANSWER:

a. $x^2 + y^2 = 841,000,000$

b.



48. **CCSS SENSE-MAKING** Suppose an unobstructed radio station broadcast could travel 120 miles.

a. Write an equation to represent the boundary of the broadcast area.

b. If the transmission tower is relocated 40 miles east and 10 miles south of the current location, and an increased signal will transmit signals an additional 80 miles, what is an equation to represent the new broadcast area?

ANSWER:

a. $x^2 + y^2 = 14,400$

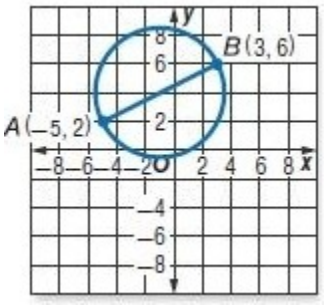
b. $(x-40)^2 + (y+10)^2 = 40,000$

49. **GEOMETRY** Concentric circles are circles with the same center but different radii. Refer to the graph at the right where $\overline{AB}$ is a diameter of the circle.

a. Write an equation of the circle concentric with the circle at the right, with radius 4 units greater.

b. Write an equation of the circle concentric with the circle at the right, with radius 2 units less.

c. Graph the circles from parts a and b on the same coordinate plane.

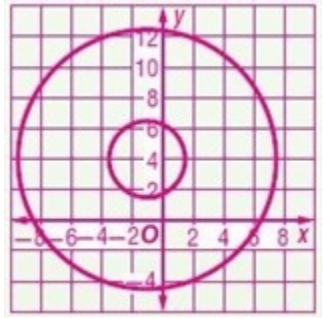


ANSWER:

a. $(x+1)^2 + (y-4)^2 = 36 + 16\sqrt{5}$

b. $(x+1)^2 + (y-4)^2 = 24 - 8\sqrt{5}$

c.



50. **EARTHQUAKES** The Rose Bowl is located about 35 miles west and about 40 miles north of downtown Los Angeles. Suppose an earthquake occurs with its epicenter about 55 miles from the stadium. Assume that the origin of a coordinate plane is located at the center of downtown Los Angeles. Write an equation for the set of points that could be the epicenter of the earthquake.

ANSWER:

$(x+35)^2 + (y-40)^2 = 3025$

CCSS PRECISION Write an equation for the circle that satisfies each set of conditions.

51. center (9, –8), passes through (19, 22)

ANSWER:

$(x-9)^2 + (y+8)^2 = 1000$

52. center $\left(-\sqrt{15}, 30\right)$, passes through the origin

ANSWER:

$\left(x+\sqrt{15}\right)^2 + (y-30)^2 = 915$

53. center at (8, –9), tangent to y-axis

ANSWER:

$(x-8)^2 + (y+9)^2 = 64$

54. center at (2, 4), tangent to x-axis

ANSWER:

$(x-2)^2 + (y-4)^2 = 16$

55. center in the first quadrant; tangent to $x = 5$, the x-axis, and the y-axis

ANSWER:

$(x-2.5)^2 + (y-2.5)^2 = 6.25$

56. center in the second quadrant; tangent to $y = 1$, $y = 5$, and the y-axis

ANSWER:

$(x+2)^2 + (y-3)^2 = 4$

57. **MULTIPLE REPRESENTATIONS** Graph $y = \sqrt{9 - x^2}$ and $y = -\sqrt{9 - x^2}$ on the same graphing calculator screen.

a. VERBAL Describe the graph formed by the union of these two graphs.

b. ALGEBRAIC Write an equation for the union of the two graphs.

c. VERBAL Most graphing calculators cannot graph the equation $x^2 + y^2 = 49$ directly. Describe a way to use a graphing calculator to graph the equation. Then graph the equation.

d. ANALYTICAL Solve $(x-2)^2 + (y+1)^2 = 4$ for y. Why do you need two equations to graph a circle on a graphing calculator?

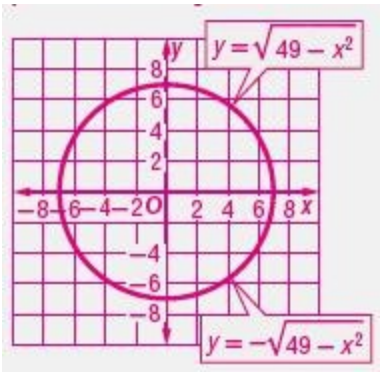
e. VERBAL Do you think that it is easier to graph the equation in part d using graph paper and a pencil or using a graphing calculator? Explain.

ANSWER:

a. circle

b. $x^2 + y^2 = 9$

c. Solve the equation for y: $y = \pm\sqrt{49 - x^2}$ .Then graph the positive and negative answers.



d. $y = \pm\sqrt{4 - (x-2)^2} - 1$; When you solve for y you must take the square root resulting in both a positive and negative answer, so you have to enter the positive equation as Y1 and the negative equation as Y2.

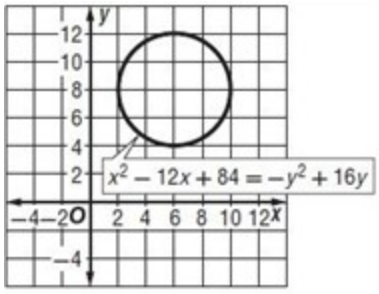
e. See students' work.

Find the center and radius of each circle. Then graph the circle.

58. $x^2 - 12x + 84 = -y^2 + 16y$

ANSWER:

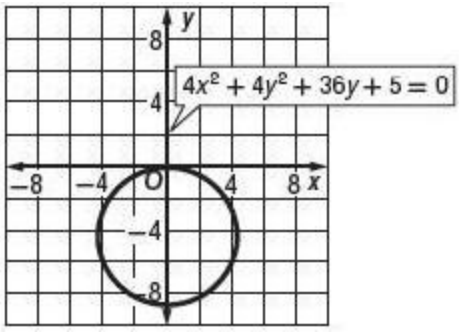
center: (6, 8), radius: 4



59. $4x^2 + 4y^2 + 36y + 5 = 0$

ANSWER:

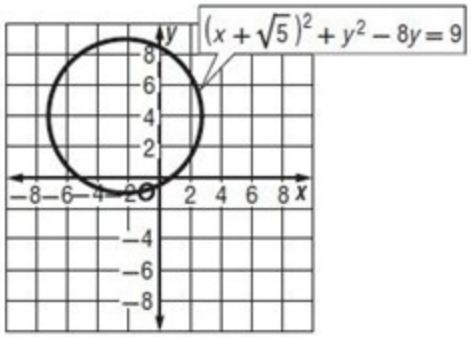
center: $\left(0, -\frac{9}{2}\right)$ radius: $\sqrt{19}$



60. $\left(x + \sqrt{5}\right)^2 + y^2 - 8y = 9$

ANSWER:

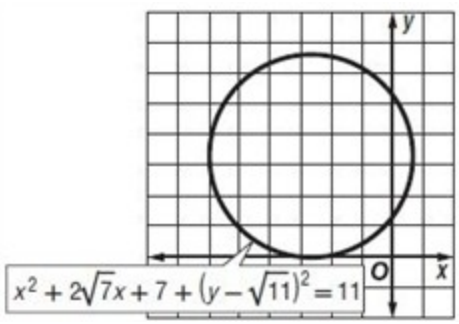
center: $\left(-\sqrt{5}, 4\right)$; radius: 5



61. $x^2 + 2\sqrt{7}x + 7 + \left(y - \sqrt{11}\right)^2 = 11$

ANSWER:

center: $\left(-\sqrt{7}, \sqrt{11}\right)$; radius: $\sqrt{11}$



62. **ERROR ANALYSIS** Heather says that $(x-2)^2 + (y+3)^2 = 36$ and $(x - 2) + (y + 3) = 6$ are equivalent equations. Carlota says that the equations are not equivalent. Is either of them correct? Explain your reasoning.

ANSWER:

Carlota; the square root of $(x-2)^2 + (y+3)^2$ is not $(x - 2) + (y + 3)$.

63. **OPEN ENDED** Consider graphs with equations of the form $(x-3)^2 + (y-a)^2 = 64$. Assign three different values for a, and graph each equation. Describe all graphs with equations of this form.

ANSWER:

See students' work; circles with a radius of 8 and centers on the graph of $x = 3$.

64. **REASONING** Explain why the phrase "in a plane" is included in the definition of a circle. What would be defined if the phrase were *not* included?

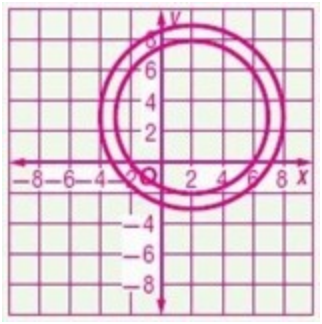
ANSWER:

If the phrase is not included, the figure would be a sphere.

65. **OPEN ENDED** Concentric circles have the same center, but most often, not the same radius. Write equations of two concentric circles. Then graph the circles.

ANSWER:

Sample answer: $(x-2)^2 + (y-3)^2 = 25$ and $(x-2)^2 + (y-3)^2 = 36$



66. **REASONING** Assume that (x, y) are the coordinates of a point on a circle. The center is at (h, k), and the radius is r. Find an equation of the circle by using the Distance Formula.

ANSWER:

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = d$$

$$\sqrt{(x-h)^2 + (y-k)^2} = r$$

$$(x-h)^2 + (y-k)^2 = r^2$$

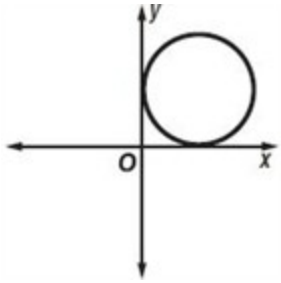
This is the standard form of the equation of a circle.

67. **WRITING IN MATH** The circle with equation $(x-a)^2 + (y-b)^2 = r^2$ lies in the first quadrant and is tangent to both the x-axis and the y-axis. Sketch the circle. Describe the possible values of a, b, and r. Do the same for a circle in Quadrants II, III, and IV. Discuss the similarities among the circles.

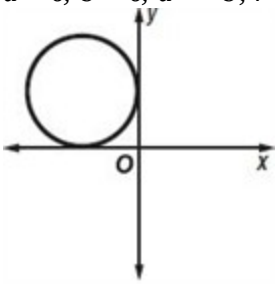
ANSWER:

Quadrant I

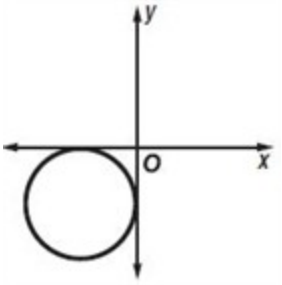
$a > 0$, $b > 0$, $a = b$, $r > 0$



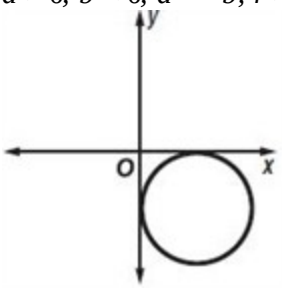
Quadrant II
$a < 0,\ b > 0,\ a = -b,\ r > 0$



Quadrant III
$a < 0,\ b < 0,\ a = b,\ r > 0$



Quadrant IV
$a > 0,\ b < 0,\ a = -b,\ r > 0$



Sample answer: The circle is rotated $90°$ about the origin from one quadrant to the next.

68. **GRIDDED RESPONSE** Two circles, both with radii 6, have exactly one point in common. If A is a point on one circle and B is a point on the other circle, what is the maximum possible length for the line segment $\overline{AB}$?

ANSWER:
24

69. In the senior class, there are 20% more girls than boys. If there are 180 girls, how many more girls than boys are there among the seniors?
A 30
B 36
C 90
D 144

ANSWER:
A

70. A $1000 deposit is made at a bank that pays 2% compounded weekly. How much will you have in your account at the end of 10 years?
F $1200.00
H $1221.36
G $1218.99
J $1224.54

ANSWER:
H

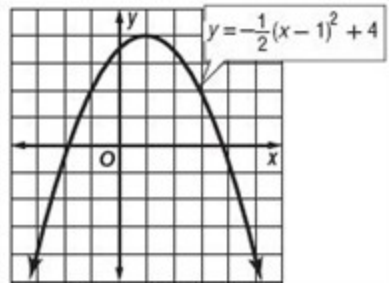
71. The mean of six numbers is 20. If one of the numbers is removed, the average of the remaining numbers is 15. What is the number that was removed?
A 42
B 43
C 45
D 48

ANSWER:
C

Graph each equation.

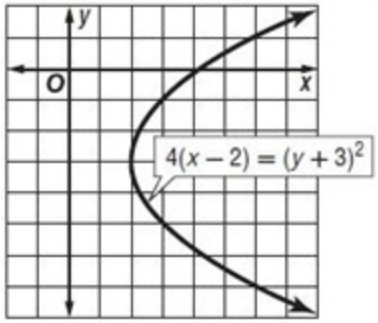
72. $y = -\frac{1}{2}(x-1)^2 + 4$

ANSWER:



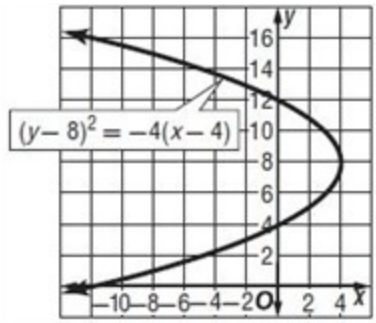
73. $4(x-2) = (y+3)^2$

ANSWER:



74. $(y-8)^2 = -4(x-4)$

ANSWER:



Find the midpoint of the line segment with endpoints at the given coordinates. Then find the distance between the points.

75. $\left(-3, -\frac{2}{11}\right), \left(5, \frac{9}{11}\right)$

ANSWER:

$\left(1, \frac{7}{22}\right); \sqrt{65}$ units

76. $\left(2\sqrt{3}, -5\right), \left(-3\sqrt{3}, 9\right)$

ANSWER:

$\left(-\frac{\sqrt{3}}{2}, 2\right); \sqrt{271}$ units

77. (2.5, 4), (–2.5, 2)

ANSWER:

(0, 3); $\sqrt{29}$ units

78. If y varies directly as x and $y = 8$ when $x = 6$, find y when $x = 15$.

ANSWER:

20

79. If y varies jointly as x and z and $y = 80$ when $x = 5$ and $z = 8$, find y when $x = 16$ and $z = 2$.

ANSWER:

64

80. If y varies inversely as x and $y = 16$ when $x = 5$, find y when $x = 20$.

ANSWER:

4

Evaluate each expression.

81. $\log_9 243$

ANSWER:

$\frac{5}{2}$

82. $\log_2 \frac{1}{32}$

ANSWER:
–5

83. $\log_3 \frac{1}{81}$

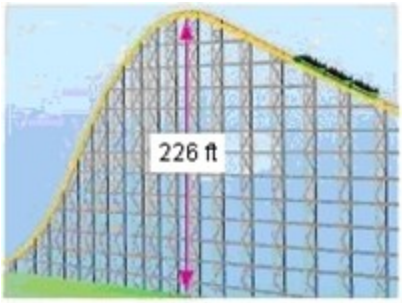
ANSWER:
–4

84. $\log_{10} 0.001$

ANSWER:
–3

85. **AMUSEMENT PARKS** The velocity *v* in feet per second of a roller coaster at the bottom of a hill is related to the vertical drop *h* in feet and the velocity v_0 in feet per second of the coaster at the top of the hill by the formula $v_0 = \sqrt{v^2 - 64h}$.

a. Explain why $v_0 = v - 8\sqrt{h}$ is not equivalent to the given formula.
b. What velocity must the coaster have at the top of the hill to achieve a velocity of 125 feet per second at the bottom?



ANSWER:
a. The square root of a difference is not the difference of the square roots.
b. 34.1 ft/s

Solve each equation by completing the square.

86. $x^2 + 3x - 18 = 0$

ANSWER:
$\{-6, 3\}$

87. $2x^2 - 3x - 3 = 0$

ANSWER:
$\left\{\frac{3 \pm \sqrt{33}}{4}\right\}$

88. $x^2 + 2x + 6 = 0$

ANSWER:
$\{-1 \pm i\sqrt{5}\}$