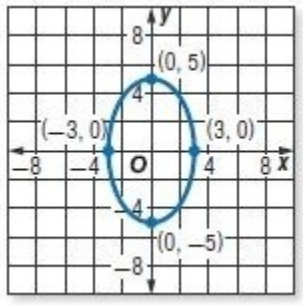
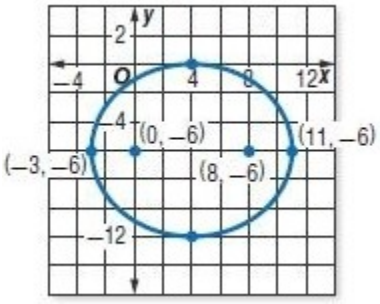
9-4 Ellipses

Write an equation of each ellipse.

1. 

ANSWER:

$$\frac{y^2}{25}+\frac{x^2}{9}=1$$

2. 

ANSWER:

$$\frac{(x-4)^2}{49}+\frac{(y+6)^2}{36}=1$$

Write an equation of an ellipse that satisfies each set of conditions.

3. vertices at (–2, –6) and (–2, 4), co-vertices at (–5, –1) and (1, –1)

ANSWER:

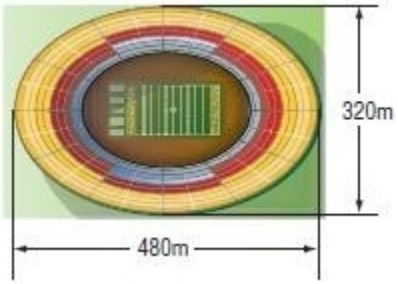
$$\frac{(y+1)^2}{25}+\frac{(x+2)^2}{9}=1$$

4. vertices at (–2, 5) and (14, 5), co-vertices at (6, 1) and (6, 9)

ANSWER:

$$\frac{(x-6)^2}{64}+\frac{(y-5)^2}{16}=1$$

5. **CCSS SENSE-MAKING** An architectural firm sent a proposal to a city for building a coliseum, shown at the right.



a. Determine the values of a and b.

b. Assuming that the center is at the origin, write an equation to represent the ellipse.

c. Determine the coordinates of the foci.

ANSWER:

a. $a = 240,\ b = 160$

b. $\frac{x^2}{57{,}600}+\frac{y^2}{25{,}600}=1.$

c. about $(179,\ 0)$ and $(-179,\ 0)$

6. **SPACE** Earth's orbit is about 91.4 million miles at perihelion and about 94.5 million miles at aphelion. Determine an equation relating Earth's orbit around the Sun in millions of miles with the center of the horizontal ellipse at the origin.

ANSWER:

$$\frac{x^2}{8720.8}+\frac{y^2}{8718.4}=1$$

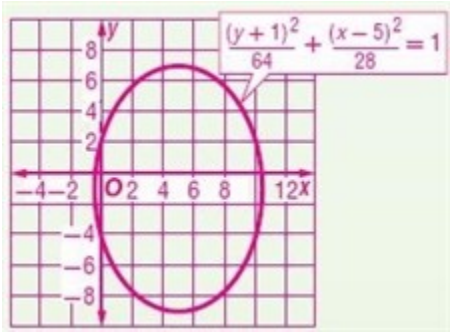
9-4 Ellipses

Find the coordinates of the center and foci and the lengths of the major and minor axes for the ellipse with the given equation. Then graph the ellipse.

7. $\frac{(y+1)^2}{64}+\frac{(x-5)^2}{28}=1$

ANSWER:

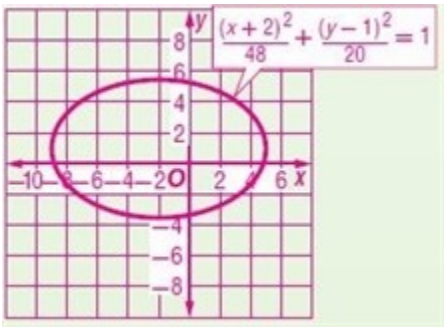
center (5, –1); foci (5, 5) and (5, –7); major axis: 16; minor axis: ≈ 10.58



8. $\frac{(x+2)^2}{48}+\frac{(y-1)^2}{20}=1$

ANSWER:

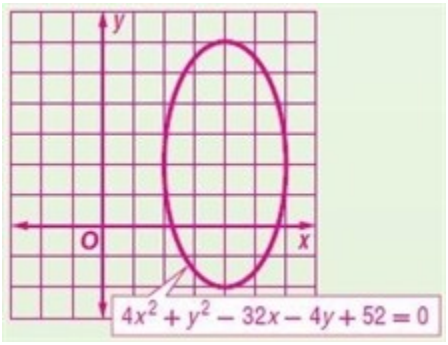
center (–2, 1); foci (3.29, 1) and (–7.29, 1); major axis: ≈ 13.86; minor axis: ≈ 8.94



9. $4x^2+y^2-32x-4y+52=0$

ANSWER:

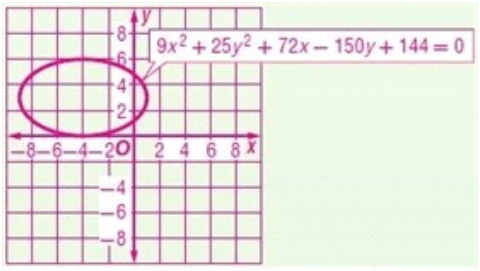
center (4, 2); foci (4, 5.46) and (4, –1.46); major axis: 8; minor axis: 4



10. $9x^2+25y^2+72x-150y+144=0$

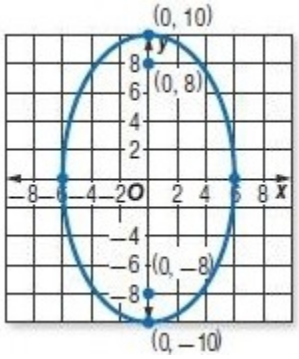
ANSWER:

center (–4, 3); foci (0, 3) and (–8, 3); major axis: 10; minor axis: 6



Write an equation of each ellipse.

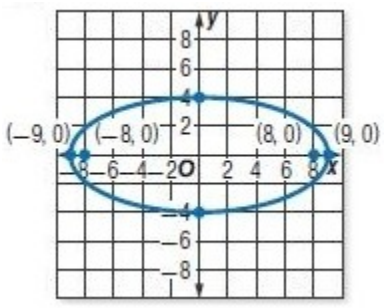
11.



ANSWER:

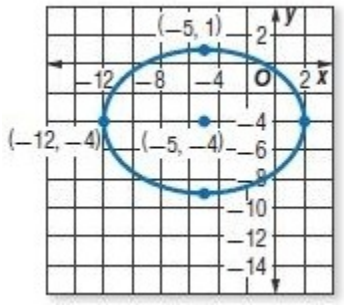
$\frac{y^2}{100}+\frac{x^2}{36}=1$

9-4 Ellipses

12. 

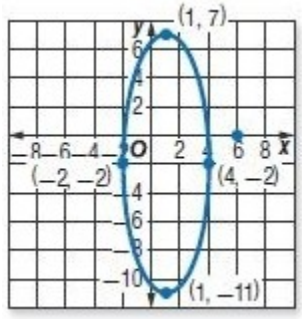
ANSWER:

$$\frac{x^2}{81}+\frac{y^2}{16}=1$$

13. 

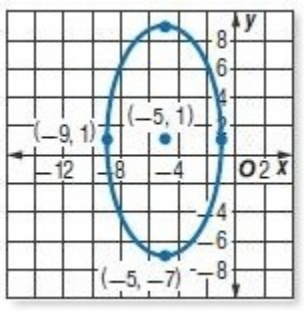
ANSWER:

$$\frac{(x+5)^2}{49}+\frac{(y+4)^2}{25}=1$$

14. 

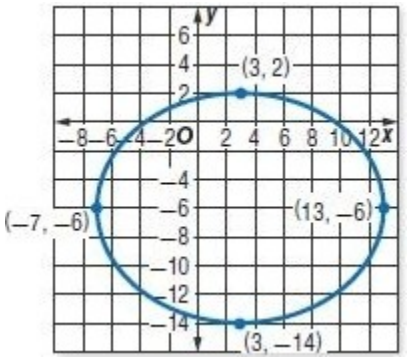
ANSWER:

$$\frac{(y+2)^2}{81}+\frac{(x-1)^2}{9}=1$$

15. 

ANSWER:

$$\frac{(y-1)^2}{64}+\frac{(x+5)^2}{16}=1$$

16. 

ANSWER:

$$\frac{(x-3)^2}{100}+\frac{(y+6)^2}{64}=1$$

Write an equation of an ellipse that satisfies each set of conditions.

17. vertices at (–6, 4) and (12, 4), co-vertices at (3, 12) and (3, –4)

ANSWER:

$$\frac{(x-3)^2}{81}+\frac{(y-4)^2}{64}=1$$

18. vertices at (–1, 11) and (–1, 1), co-vertices at (–4, 6) and (2, 6)

ANSWER:

$$\frac{(y-6)^2}{25}+\frac{(x+1)^2}{9}=1$$

19. center at (–2, 6), vertex at (–2, 16), co-vertex at (1, 6)

ANSWER:

$$\frac{(y-6)^2}{100}+\frac{(x+2)^2}{9}=1$$

20. center at (3, –4), vertex at (8, –4), co-vertex at (3, –2)

ANSWER:

$$\frac{(x-3)^2}{25}+\frac{(y+4)^2}{4}=1$$

21. vertices at (4, 12) and (4, –4), co-vertices at (1, 4) and (7, 4)

ANSWER:

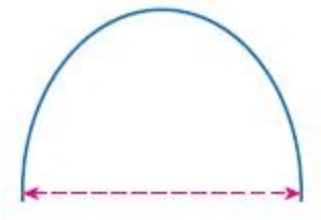
$$\frac{(y-4)^2}{64}+\frac{(x-4)^2}{9}=1$$

22. vertices at (–11, 2) and (–1, 2), co-vertices at (–6, 0) and (–6, 4)

ANSWER:

$$\frac{(x+6)^2}{25}+\frac{(y-2)^2}{4}=1$$

23. **CCSS MODELING** The opening of a tunnel in the mountains can be modeled by semiellipses, or halves of ellipses. If the opening is 14.6 meters wide and 8.6 meters high, determine an equation to represent the opening with the center at the origin.
Refer to the photo on Page 620.



ANSWER:

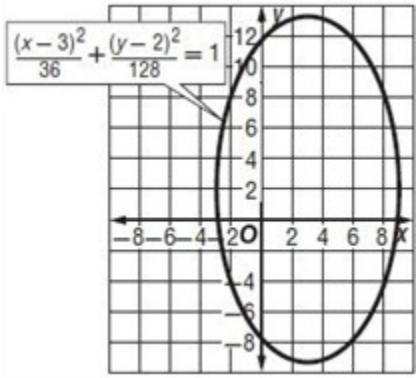
$$\frac{y^2}{73.96}+\frac{x^2}{53.29}=1$$

Find the coordinates of the center and foci and the lengths of the major and minor axes for the ellipse with the given equation. Then graph the ellipse.

24. $\frac{(x-3)^2}{36}+\frac{(y-2)^2}{128}=1$

ANSWER:

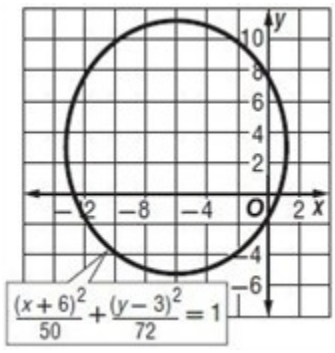
center (3, 2); foci (3, 11.59) and (3, –7.59); major axis: ≈ 22.63; minor axis: 12



25. $\frac{(x+6)^2}{50}+\frac{(y-3)^2}{72}=1$

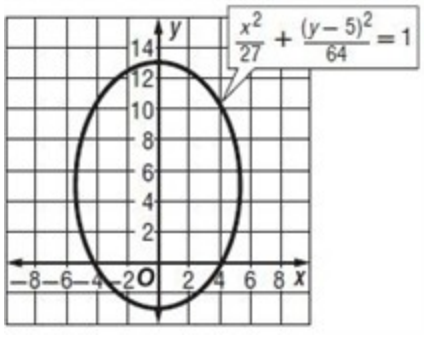
ANSWER:

center (–6, 3); foci (–6, 7.69) and (–6, –1.69); major axis: ≈ 16.97; minor axis: ≈ 14.14



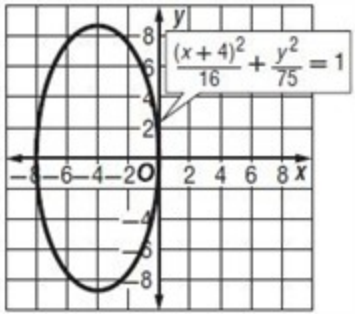
26. $\frac{x^2}{27}+\frac{(y-5)^2}{64}=1$

ANSWER:
center (0, 5); foci (0, 11.08) and (0, −1.08); major axis: 16; minor axis: ≈ 10.39



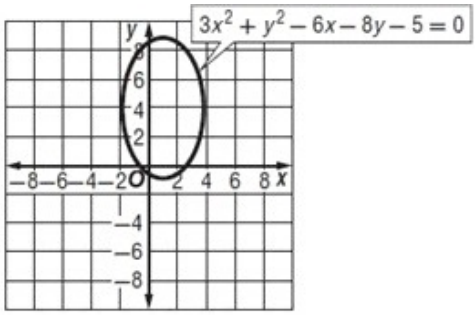
27. $\frac{(x+4)^2}{16}+\frac{y^2}{75}=1$

ANSWER:
center (−4, 0); foci (−4, 7.68) and (−4, −7.68); major axis: ≈ 17.32; minor axis: 8



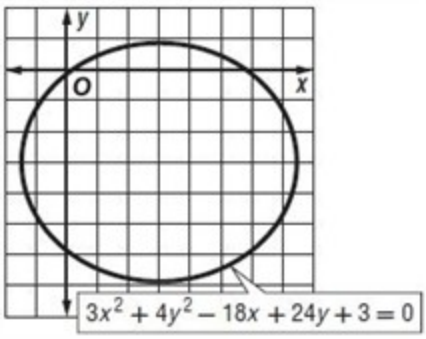
28. $3x^2+y^2-6x-8y-5=0$

ANSWER:
center (1, 4); foci (1, 8) and (1, 0); major axis: ≈ 9.80; minor axis: ≈ 5.66



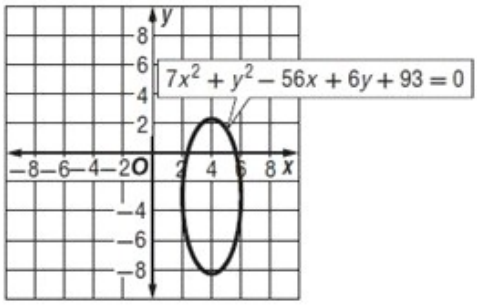
29. $3x^2+4y^2-18x+24y+3=0$

ANSWER:
center (3, −3); foci (5.24, −3) and (0.76, −3); major axis: ≈ 8.94; minor axis: ≈ 7.75



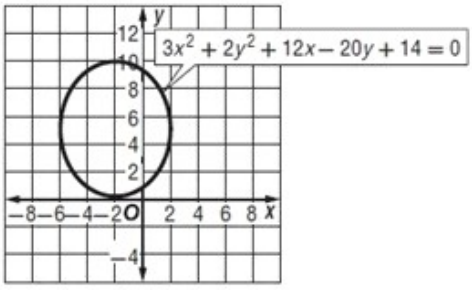
30. $7x^2+y^2-56x+6y+93=0$

ANSWER:
center (4, −3); foci (4, 1.90) and (4, −7.90); major axis: ≈ 10.58; minor axis: 4



31. $3x^2+2y^2+12x-20y+14=0$

ANSWER:
center (−2, 5); foci (−2, 7.83) and (−2, 2.17); major axis: ≈ 9.80; minor axis: 8



32. **SPACE** Like the planets, Halley's Comet travels around the Sun in an elliptical orbit. The aphelion is 3282.9 million miles and the perihelion is 54.87 million miles. Determine an equation relating the comet's orbit around the Sun in millions of miles with the center of the horizontal ellipse at the origin.

ANSWER:

$$\frac{x^2}{2,786,629.3}+\frac{y^2}{181,584.8}=1$$

Write an equation of an ellipse that satisfies each set of conditions.

33. center at (–5, –2), focus at (–5, 2), co-vertex at (–8, –2)

ANSWER:

$$\frac{(y+2)^2}{25}+\frac{(x+5)^2}{9}=1$$

34. center at (4, –3), focus at (9, –3), co-vertex at (4, –5)

ANSWER:

$$\frac{(x-4)^2}{29}+\frac{(y+3)^2}{4}=1$$

35. foci at (–2, 8) and (6, 8), co-vertex at (2, 10)

ANSWER:

$$\frac{(x-2)^2}{20}+\frac{(y-8)^2}{4}=1$$

36. foci at (4, 4) and (4, 14), co-vertex at (0, 9)

ANSWER:

$$\frac{(y-9)^2}{41}+\frac{(x-4)^2}{16}=1$$

37. **GOVERNMENT** The Oval Office is located in the West Wing of the White House. It is an elliptical shaped room used as the main office by the President of the United States. The long axis is 10.9 meters long and the short axis is 8.8 meters long. Write an equation to represent the outer walls of the Oval Office. Assume that the center of the room is at the origin.

ANSWER:

$$\frac{x^2}{29.7025}+\frac{y^2}{19.36}=1 \text{ or } \frac{y^2}{29.7025}+\frac{x^2}{19.36}=1$$

38. **SOUND** A whispering gallery is an elliptical room in which a faint whisper at one focus that cannot be heard by other people in the room, can easily be heard by someone at the other focus. Suppose an ellipse is 400 feet long and 120 feet wide. What is the distance between the foci?

ANSWER:

about 381.58 ft

39. **MULTIPLE REPRESENTATIONS** The *eccentricity* of an ellipse measures how circular the ellipse is.

a. GRAPHICAL Graph $\frac{x^2}{81}+\frac{y^2}{36}=1$ **and** $\frac{x^2}{81}+\frac{y^2}{9}=1$ on the same graph.

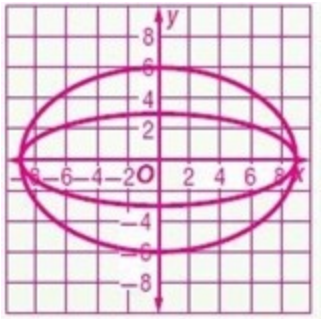
b. VERBAL Describe the difference between the two graphs.

c. ALGEBRAIC The eccentricity of an ellipse is $\frac{c}{a}$. Find the eccentricity for each.

d. ANALYTICAL Make a conjecture about the relationship between the value of an ellipse's eccentricity and the shape of the ellipse as compared to a circle.

ANSWER:

a.



b. Sample answer: The first graph is more circular than the second graph.

c. first graph: 0.745; second graph: 0.943

d. Sample answer: The closer the eccentricity is to 0, the more circular the ellipse.

40. **ERROR ANALYSIS** Serena and Karissa are determining the equation of an ellipse with foci at (–4, –11) and (–4, 5) and co-vertices at (2, –3) and (–10, –3). Is either of them correct? Explain your reasoning.

Serena

$$\frac{(x-4)^2}{64}+\frac{(y+3)^2}{36}=1$$

Karissa

$$\frac{(x+4)^2}{100}+\frac{(y+3)^2}{36}=1$$

ANSWER:

Sample answer: Neither; both are showing horizontal ellipses and answer is vertical.

41. **OPEN ENDED** Write an equation of an ellipse with a focus at the origin.

ANSWER:

Sample answer: $\frac{(x+4)^2}{40}+\frac{y^2}{24}=1$

42. **CHALLENGE** When the values of a and b are equal, an ellipse is a circle. Use this information and your knowledge of ellipses to determine the formula for the area of an ellipse in terms of a and b.

ANSWER:

$A=\pi ab$

43. **CHALLENGE** Determine an equation of an ellipse with foci at $\left(2,\sqrt{6}\right)$ and $\left(2,-\sqrt{6}\right)$ that passes through $\left(3,\sqrt{6}\right)$.

ANSWER:

$\frac{y^2}{9}+\frac{(x-2)^2}{3}=1$

44. **CCSS ARGUMENTS** What happens to the location of the foci as an ellipse becomes more circular? Explain your reasoning.

ANSWER:

Sample answer: As an ellipse becomes more circular, the difference between *a* and *b* becomes smaller. This causes the value of *c* to become smaller since $c^2 = a^2 - b^2$. The value of 2*c* is the distance between the foci, so the foci get closer together.

45. **REASONING** An ellipse has foci at (–7, 2) and (18, 2). If (2, 14) is a point on the ellipse, show that (2, –10) is also a point on the ellipse.

ANSWER:

For any point on an ellipse, the sum of the distances from that point to the foci is constant by the definition of an ellipse. So, if (2, 14) is on the ellipse, then the sum of the distances from it to the foci will be a certain value consistent with every other point on the ellipse.

The distance between (–7, 2) and (2, 14) is $\sqrt{(-7-2)^2+(2-14)^2}$ or 15.
The distance between (18, 2) and (2, 14) is $\sqrt{(18-2)^2+(2-14)^2}$ or 20.

The sum of these two distances is 35.
The distance between (–7, 2) and (2, –10) is $\sqrt{(-7-2)^2+[2-(-10)]^2}$ or 15.
The distance between (18, 2) and (2, –10) is $\sqrt{(18-2)^2+[2-(-10)]^2}$ or 20.

The sum of these distances is also 35. Thus, (2, –10) also lies on the ellipse.

46. **WRITING IN MATH** Explain why the domain is $\{x|-a \le x \le a\}$ and the range is $\{y|-b \le y \le b\}$ for an ellipse with equation $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

ANSWER:

Sample answer:
The domain is $(x|-a \le x \le a)$ because, if $|x| > a$, then $\frac{x^2}{a^2}$ would be greater than 1.

This will force $\frac{y^2}{b^2}$ to be negative since $\frac{x^2}{a^2}+\frac{y^2}{b^2}$ must equal 1.

In order for $\frac{y^2}{b^2}$ to be negative, either y^2 or b^2 must be negative, which cannot happen with real numbers.

For any values of $\{x|-a \le x \le a\}$, $\frac{x^2}{a^2}$ will be between 0 and 1.

The value of $\frac{y^2}{b^2}$ will also be between 0 and 1 for $(y|-b \le y \le b)$ and there are infinite combinations of $\frac{x^2}{a^2}$ and $\frac{y^2}{b^2}$ for which $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

Thus, the domain must be $(x|-a \le x \le a)$. The same method proves that the range must be $(y|-b \le y \le b)$.

47. Multiply.

$$(2 + 3i)(4 + 7i)$$

A $8 + 21i$
B $-13 + 26i$
C $-6 + 10i$
D $13 + 12i$

ANSWER:

B

48. The average lifespan of American women has been tracked, and the model for the data is $y = 0.2t + 73$, where $t = 0$ corresponds to 1960. What is the meaning of the y-intercept?
F In 2007, the average lifespan was 60.
G In 1960, the average lifespan was 58.
H In 1960, the average lifespan was 73.
J The lifespan is increasing 0.2 years every year

ANSWER:
H

49. **GRIDDED RESPONSE** If we decrease a number by 6 and then double the result, we get 5 less than the number. What is the number?

ANSWER:
7

50. **SAT/ACT** The length of a rectangular prism is one inch greater than its width. The height is three times the length. Find the volume of the prism.
A $3x^3 + x^2 + 3x$
B $x^3 + x^2 + x$
C $3x^3 + 6x^2 + 3x$
D $3x^3 + 3x^2 + 3x$
E $3x^3 + 3x^2$

ANSWER:
C

Write an equation of the circle that satisfies each set of conditions.

51. center (8, –9), passes through (21, 22)

ANSWER:
$(x-8)^2 + (y+9)^2 = 1130$

52. center at (4, 2), tangent to x-axis

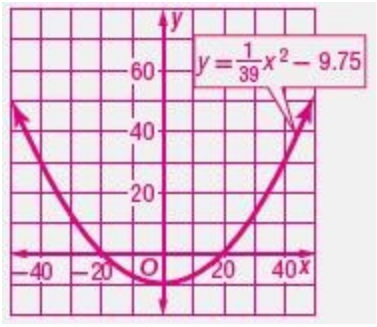
ANSWER:
$(x-4)^2 + (y-2)^2 = 4$

53. center in the second quadrant; tangent to $y = –1$, $y = 9$, and the y-axis

ANSWER:
$(x+5)^2 + (y-4)^2 = 25$

54. **ENERGY** A parabolic mirror is used to collect solar energy. The mirrors reflect the rays from the Sun to the focus of the parabola. The focus of a particular mirror is 9.75 feet above the vertex, and the latus rectum is 39 feet long.
a. Assume that the focus is at the origin. Write an equation of the parabola formed by the mirror.
b. One foot is exactly 0.3048 meter. Rewrite the equation of the mirror in meters.
c. Graph one of the equations for the mirror.
d. Which equation did you choose to graph? Explain why.

ANSWER:
a. $y = \frac{1}{39}x^2 - 9.75$
b. $y = \frac{1}{11.8872}x^2 - 2.9718$
c.



d. Sample answer: The equation in feet is easier to graph because the numbers have fewer decimal places.

Simplify each expression.

55. $\frac{6}{d^2 + 4d + 4} + \frac{5}{d+2}$

ANSWER:
$\frac{5d+16}{(d+2)^2}$

56. $\frac{a}{a^2 - a - 20} + \frac{2}{a+4}$

ANSWER:
$\frac{3a-10}{(a-5)(a+4)}$

57. $\dfrac{x}{x+1}+\dfrac{3}{x^2-4x-5}$

ANSWER:

$\dfrac{x^2-5x+3}{(x-5)(x+1)}$

Solve each equation.

58. $\log_{10}\left(x^2+1\right)=1$

ANSWER:

± 3

59. $\log_b 64=3$

ANSWER:

4

60. $\log_b 121=2$

ANSWER:

11

Simplify.

61. $-5ab^2\left(-3a^2b+6a^3b-3a^4b^4\right)$

ANSWER:

$15a^3b^3-30a^4b^3+15a^5b^6$

62. $2xy\left(3xy^3-4xy+2y^4\right)$

ANSWER:

$6x^2y^4-8x^2y^2+4xy^5$

63. $\left(4x^2-3y^2+5xy\right)-\left(8xy+3y^2\right)$

ANSWER:

$4x^2-3xy-6y^2$

64. $\left(10x^2-3xy+4y^2\right)-\left(3x^2+5xy\right)$

ANSWER:

$7x^2-8xy+4y^2$

Write an equation of the line passing through each pair of points.

65. (–2, 5) and (3, 1)

ANSWER:

$y=-\dfrac{4}{5}x+\dfrac{17}{5}$

66. (7, 1) and (7, 8)

ANSWER:

$x = 7$

67. (–3, 5) and (2, 2)

ANSWER:

$y=-\dfrac{3}{5}x+\dfrac{16}{5}$