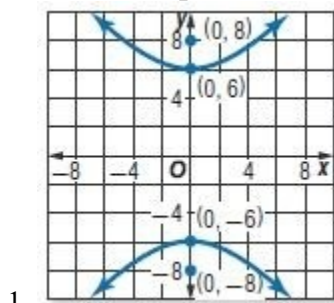


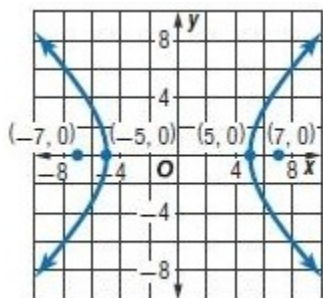
9-5 Hyperbolas

Write an equation for each hyperbola.



ANSWER:

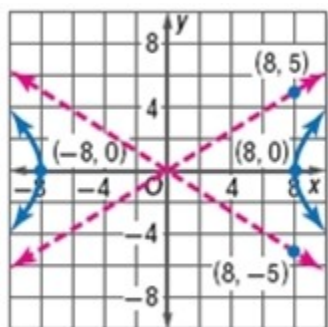
$$\frac{y^2}{36} - \frac{x^2}{28} = 1$$



2.

ANSWER:

$$\frac{x^2}{25} - \frac{y^2}{24} = 1$$

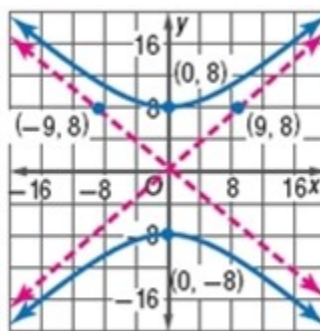


3.

ANSWER:

$$\frac{x^2}{64} - \frac{y^2}{25} = 1$$

4.



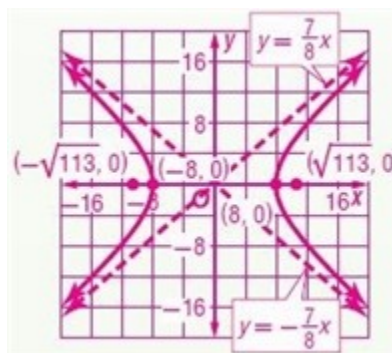
ANSWER:

$$\frac{y^2}{64} - \frac{x^2}{81} = 1$$

CCSS STRUCTURE Graph each hyperbola. Identify the vertices, foci, and asymptotes.

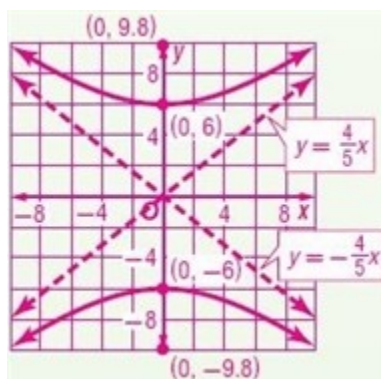
5. $\frac{x^2}{64} - \frac{y^2}{49} = 1$

ANSWER:



6. $\frac{y^2}{36} - \frac{x^2}{60} = 1$

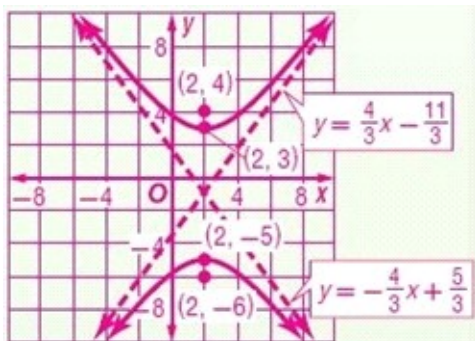
ANSWER:



9-5 Hyperbolas

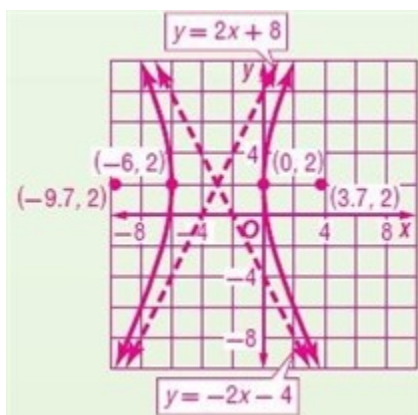
7. $9y^2 + 18y - 16x^2 + 64x - 199 = 0$

ANSWER:



8. $4x^2 + 24x - y^2 + 4y - 4 = 0$

ANSWER:

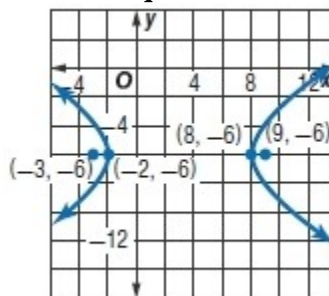


9. **NAVIGATION** A ship determines that the difference of its distances from two stations is 60 nautical miles. Write an equation for a hyperbola on which the ship lies if the stations are at $(-80, 0)$ and $(80, 0)$.

ANSWER:

$$\frac{x^2}{900} - \frac{y^2}{5500} = 1$$

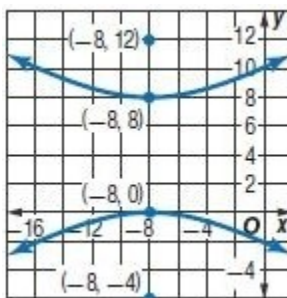
Write an equation for each hyperbola.



10.

ANSWER:

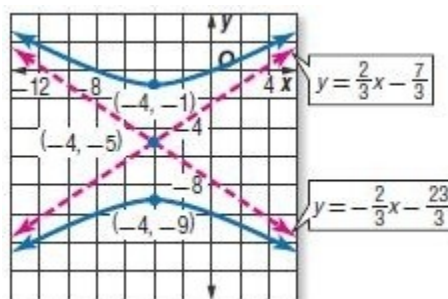
$$\frac{(x-3)^2}{25} - \frac{(y+6)^2}{11} = 1$$



11.

ANSWER:

$$\frac{(y-4)^2}{16} - \frac{(x+8)^2}{48} = 1$$

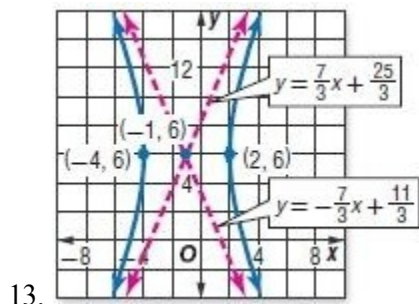


12.

ANSWER:

$$\frac{(y+5)^2}{16} - \frac{(x+4)^2}{36} = 1$$

9-5 Hyperbolas



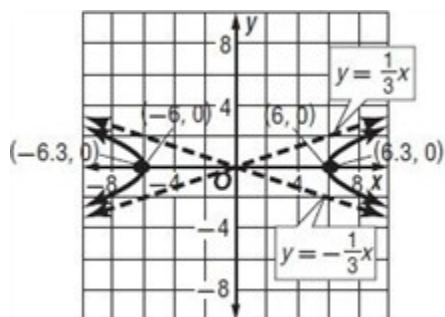
ANSWER:

$$\frac{(x+1)^2}{9} - \frac{(y-6)^2}{49} = 1$$

Graph each hyperbola. Identify the vertices, foci, and asymptotes.

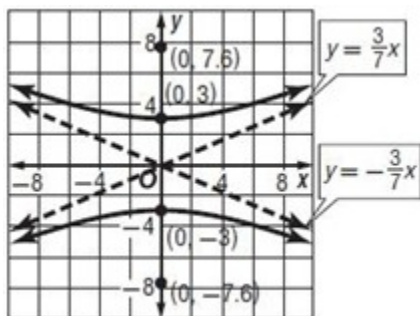
14. $\frac{x^2}{36} - \frac{y^2}{4} = 1$

ANSWER:



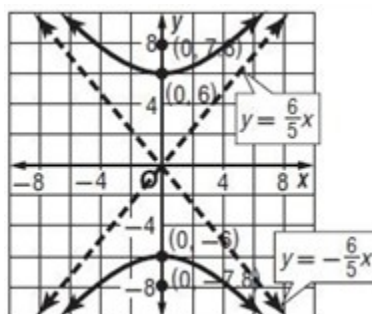
15. $\frac{y^2}{9} - \frac{x^2}{49} = 1$

ANSWER:



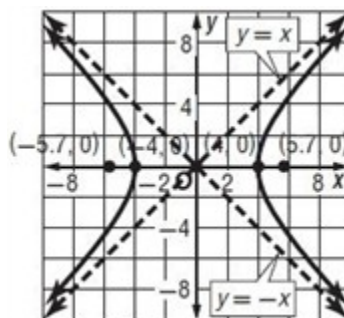
16. $\frac{y^2}{36} - \frac{x^2}{25} = 1$

ANSWER:



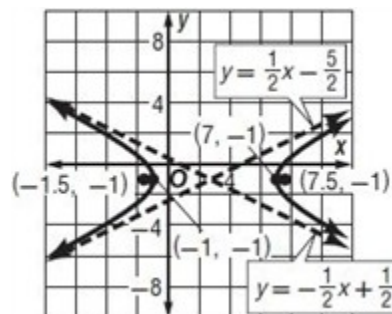
17. $\frac{x^2}{16} - \frac{y^2}{16} = 1$

ANSWER:



18. $\frac{(x-3)^2}{16} - \frac{(y+1)^2}{4} = 1$

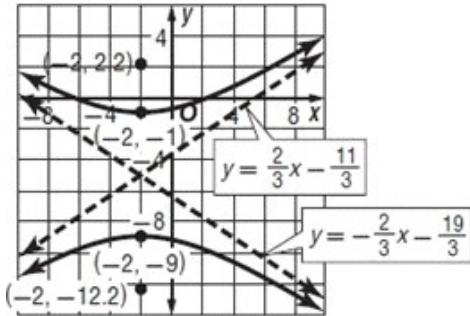
ANSWER:



9-5 Hyperbolas

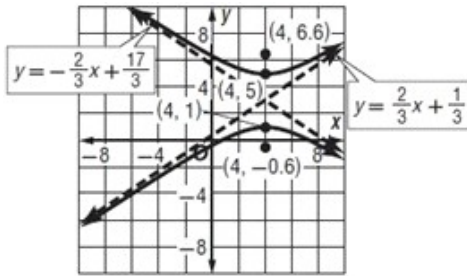
19. $\frac{(y+5)^2}{16} - \frac{(x+2)^2}{36} = 1$

ANSWER:



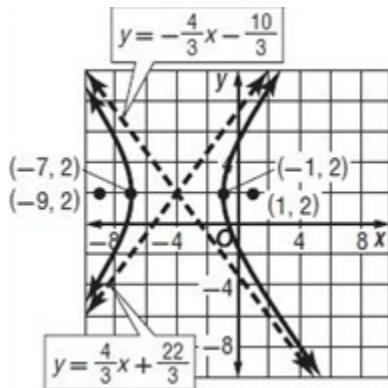
20. $9y^2 - 4x^2 - 54y + 32x - 19 = 0$

ANSWER:



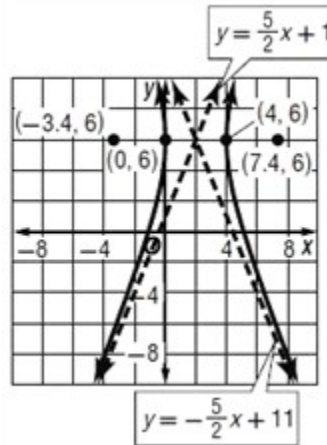
21. $16x^2 - 9y^2 + 128x + 36y + 76 = 0$

ANSWER:



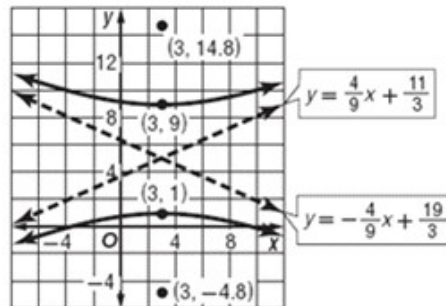
22. $25x^2 - 4y^2 - 100x + 48y - 144 = 0$

ANSWER:



23. $81y^2 - 16x^2 - 810y + 96x + 585 = 0$

ANSWER:



24. **NAVIGATION** A ship determines that the difference of its distances from two stations is 80 nautical miles. Write an equation for a hyperbola on which the ship lies if the stations are at $(-100, 0)$ and $(100, 0)$.

ANSWER:

$$\frac{x^2}{1600} - \frac{y^2}{8400} = 1$$

Determine whether the following equations represent ellipses or hyperbolas.

25. $4x^2 = 5y^2 + 6$

ANSWER:

hyperbola

9-5 Hyperbolas

26. $8x^2 - 2x = 8y - 3y^2$

ANSWER:

ellipse

27. $-5x^2 + 4x = 6y + 3y^2$

ANSWER:

ellipse

28. $7y - 2x^2 = 6x - 2y^2$

ANSWER:

hyperbola

29. $6x - 7x^2 - 5y^2 = 2y$

ANSWER:

ellipse

30. $4x + 6y + 2x^2 = -3y^2$

ANSWER:

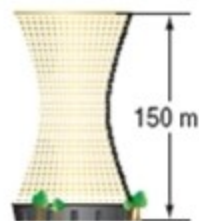
ellipse

31. **SPACE** Refer to the application at the beginning of the lesson. With the Sun as a focus and the center at the origin, a certain comet's path follows a branch of a hyperbola. If two of the coordinates of the path are (10, 0) and (30, 100) where the units are in millions of miles, determine the equation of the path.

ANSWER:

$$\frac{x^2}{100} - \frac{y^2}{1250} = 1$$

32. **COOLING** Natural draft cooling towers are shaped like hyperbolas for more efficient cooling of power plants. The hyperbola in the tower can be modeled by $\frac{x^2}{16} - \frac{y^2}{225} = 1$, where the units are in meters. Find the width of the tower at the top and at its narrowest point in the middle.



ANSWER:

8 m in the middle and 40.8 m at the top

33. **MULTIPLE REPRESENTATIONS** Consider $xy = 16$.
- TABULAR** Make a table of values for the equation for $-12 \leq x \leq 12$.
 - GRAPHICAL** Graph the hyperbola represented by the equation.
 - LOGICAL** Determine and graph the asymptotes for the hyperbola.
 - ANALYTICAL** What special property do you notice about the asymptotes? Hyperbolas that represent this property are called *rectangular hyperbolas*.
 - ANALYTICAL** Without any calculations, what do you think will be the coordinates of the vertices for $xy = 25$? for $xy = 36$?

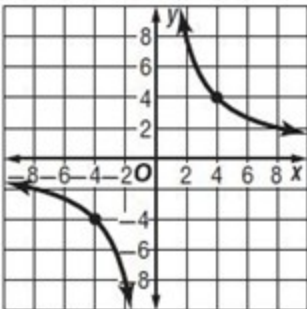
ANSWER:

a.

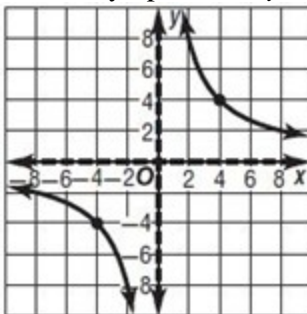
9-5 Hyperbolas

x	y
-12	-1.33
-10	-1.6
-8	-2
-6	-2.67
-4	-4
-2	-8
0	undef
2	8
4	4
6	2.67
8	2
10	1.6
12	1.33

b.



c. The asymptotes are $y = 0$ and $x = 0$.



d. They are perpendicular.

e. For $xy = 25$, the vertices will be at $(5, 5)$ and $(-5, -5)$, and for $xy = 36$, they will be at $(-6, -6)$ and $(6, 6)$.

34. **CCSS MODELING** Two receiving stations that are 250 miles apart receive a signal from a downed airplane. They determine that the airplane is 70 miles farther from station B than from station A . Determine the equation of the horizontal hyperbola centered at the origin on which the plane is located.

ANSWER:

$$\frac{x^2}{1225} - \frac{y^2}{14400} = 1$$

35. **WEATHER** Luisa and Karl live exactly 4000 feet apart. While on the phone at their homes, Luisa hears thunder out of her window and Karl hears it 3 seconds later out of his. If sound travels 1100 feet per second, determine the equation for the horizontal hyperbola on which the lightning is located.

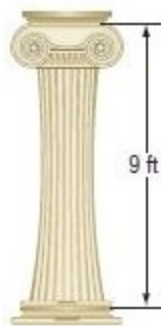
ANSWER:

$$\frac{x^2}{2,722,500} - \frac{y^2}{1,277,500} = 1$$

36. **ARCHITECTURE** Large pillars with cross sections in the shape of hyperbolas were popular in ancient Greece. The curves can be modeled by the

equation $\frac{x^2}{0.16} - \frac{y^2}{4} = 1$, where the units are in feet.

If the pillars are 9 feet tall, find the width of the top of each pillar and the width of each pillar at the narrowest point in the middle. Round to the nearest hundredth of a foot.



ANSWER:

≈ 1.96 ft, 0.8 ft

Write an equation for the hyperbola that satisfies each set of conditions.

37. vertices $(-8, 0)$ and $(8, 0)$, conjugate axis of length 20 units

ANSWER:

$$\frac{x^2}{64} - \frac{y^2}{100} = 1$$

38. vertices $(0, -6)$ and $(0, 6)$, conjugate axis of length 24 units

ANSWER:

$$\frac{y^2}{36} - \frac{x^2}{144} = 1$$

9-5 Hyperbolas

39. vertices $(6, -2)$ and $(-2, -2)$, foci $(10, -2)$ and $(-6, -2)$

ANSWER:

$$\frac{(x-2)^2}{16} - \frac{(y+2)^2}{48} = 1$$

40. vertices $(-3, 4)$ and $(-3, -8)$, foci $(-3, 9)$ and $(-3, -13)$

ANSWER:

$$\frac{(y+2)^2}{36} - \frac{(x+3)^2}{85} = 1$$

41. centered at the origin with a horizontal transverse axis of length 10 units and a conjugate axis of length 4 units

ANSWER:

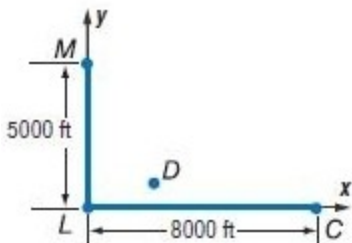
$$\frac{x^2}{25} - \frac{y^2}{4} = 1$$

42. centered at the origin with a vertical transverse axis of length 16 units and a conjugate axis of length 12 units

ANSWER:

$$\frac{y^2}{64} - \frac{x^2}{36} = 1$$

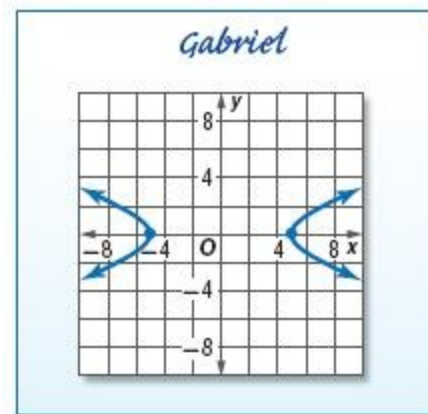
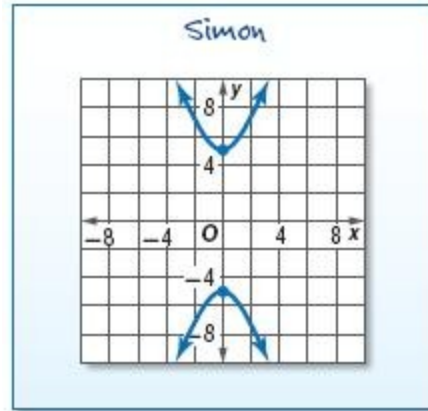
43. **TRIANGULATION** While looking for their lost dog in the woods, Lae, Meg, and Cesar hear a bark. Meg hears it 2 seconds before Lae and Cesar hears it 3 seconds before Lae. With Lae at the origin, determine the exact location of their dog if sound travels 1100 feet per second.



ANSWER:

$(2308, 826)$

44. **CCSS CRITIQUE** Simon and Gabriel are graphing $\frac{y^2}{25} - \frac{x^2}{4} = 1$. Is either of them correct? Explain your reasoning.



ANSWER:

Sample answer: Simon; The equation indicates a hyperbola that opens up and down. Gabriel drew a horizontal hyperbola.

45. **CHALLENGE** The origin lies on a horizontal hyperbola. The asymptotes for the hyperbola are $y = -x + 1$ and $y = x - 5$. Find the equation for the hyperbola.

ANSWER:

$$\frac{(x-3)^2}{5} - \frac{(y+2)^2}{5} = 1$$

9-5 Hyperbolas

46. **REASONING** What happens to the location of the foci of a hyperbola as the value of a becomes increasingly smaller than the value of b ? Explain your reasoning.

ANSWER:

Sample answer: The foci move farther away from the vertices. When a is only slightly smaller than b , the value of c is only fairly larger than a . However, as b becomes much larger than a , by way of $c^2 = a^2 + b^2$, c becomes much larger than a . When this happens, the distance between the foci c becomes much greater than the distance between the vertices a . Therefore, the foci are much farther away from the vertices.

47. **REASONING** Consider $\frac{y^2}{36} - \frac{x^2}{16} = 1$. Describe the change in the shape of the hyperbola and the locations of the vertices and foci if 36 is changed to 9. Explain why this happens.

ANSWER:

Sample answer: When 36 changes to 9, the vertical hyperbola widens (splits out from the y -axis faster). This is due to a smaller value of y being needed to produce the same value of x . The vertices are moved closer together due to the value of a decreasing from 6 to 3. The foci move farther from the vertices because the difference between c and a increased.

48. **OPEN ENDED** Write an equation for a hyperbola with a focus at the origin.

ANSWER:

Sample answer: $\frac{(x-5)^2}{9} - \frac{y^2}{16} = 1$

49. **WRITING IN MATH** Why would you choose a conic section to model a set of data instead of a polynomial function?

ANSWER:

Sample answer: Conic sections can be used to model phenomena that can't be modeled using functions. For example, parabolas can be used to model paths of comets and ellipses can be used to model planetary orbits.

50. You have 6 more dimes than quarters. You have a total of \$5.15. How many dimes do you have?

A 13
B 16
C 19
D 25

ANSWER:

C

51. How tall is a tree that is 15 feet shorter than a pole three times as tall as the tree?

F 24.5 ft
G 22.5 ft
H 21.5 ft
J 7.5 ft

ANSWER:

J

52. **SHORT RESPONSE** A rectangle is 8 feet long and 6 feet wide. If each dimension is increased by the same number of feet, the area of the new rectangle formed is 32 square feet more than the area of the original rectangle. By how many feet was each dimension increased?

ANSWER:

2

53. **SAT/ACT** When the equation $y = 4x^2 - 5$ is graphed in the coordinate plane, the graph is which of the following?

A line
B circle
C ellipse
D hyperbola
E parabola

ANSWER:

E

Write an equation for an ellipse that satisfies each set of conditions.

54. endpoints of major axis at (2, 2) and (2, -10), endpoints of minor axis at (0, -4) and (4, -4)

ANSWER:

$$\frac{(y+4)^2}{36} + \frac{(x-2)^2}{4} = 1$$

9-5 Hyperbolas

55. endpoints of major axis at $(0, 10)$ and $(0, -10)$, foci at $(0, 8)$ and $(0, -8)$

ANSWER:

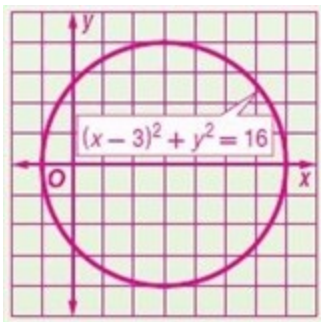
$$\frac{y^2}{100} + \frac{x^2}{36} = 1$$

Find the center and radius of the circle with the given equation. Then graph the circle.

56. $(x-3)^2 + y^2 = 16$

ANSWER:

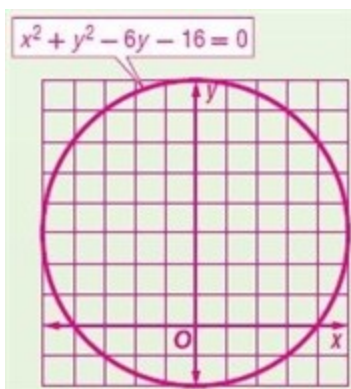
$(3, 0)$, 4 units



57. $x^2 + y^2 - 6y - 16 = 0$

ANSWER:

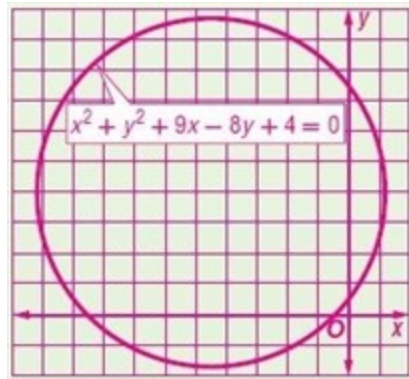
$(0, 3)$, 5 units



58. $x^2 + y^2 + 9x - 8y + 4 = 0$

ANSWER:

$$\left(-\frac{9}{2}, 4\right), \frac{\sqrt{129}}{2} \text{ units}$$



9-5 Hyperbolas

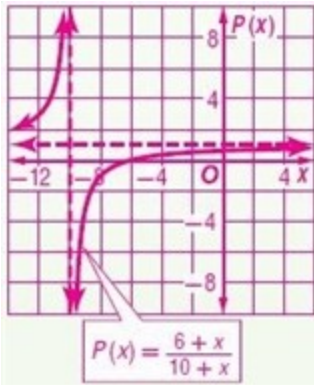
59. **BASKETBALL** Zonta plays basketball for Centerville High School. So far this season, she has made 6 out of 10 free-throws. She is determined to improve her free-throw percentage. If she can make x consecutive free throws, her free-throw

percentage can be determined using $P(x) = \frac{6+x}{10+x}$.

- Graph the function.
- What part of the graph is meaningful in the context of the problem?
- Describe the meaning of the y -intercept.
- What is the equation of the horizontal asymptote? Explain its meaning with respect to Zonta's shooting percentage.

ANSWER:

a.



- the part in the first quadrant
- It represents her original free-throw percentage of 60%.
- $P(x) = 1$; this represents 100%, which she cannot achieve because she has already missed 4 free throws.

Solve each equation.

60. $\left(\frac{1}{7}\right)^{y-3} = 343$

ANSWER:

0

61. $10^{x-1} = 100^{2x-3}$

ANSWER:

$\frac{5}{3}$

62. $36^{2p} = 216^{p-1}$

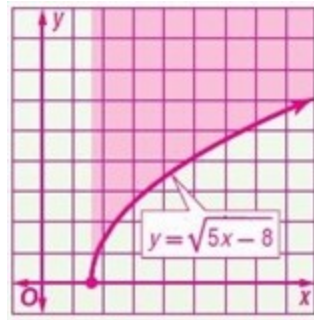
ANSWER:

-3

Graph each inequality.

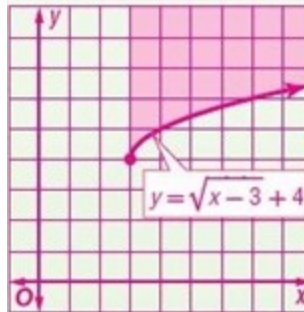
63. $y \geq \sqrt{5x-8}$

ANSWER:



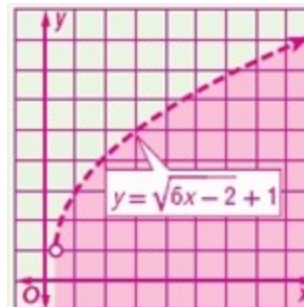
64. $y \geq \sqrt{x-3} + 4$

ANSWER:



65. $y < \sqrt{6x-2} + 1$

ANSWER:



66. Write an equation for a parabola with vertex at the origin that passes through $(2, -8)$.

ANSWER:

$y = -2x^2$

9-5 Hyperbolas

67. Write an equation for a parabola with vertex at $(-3, -4)$ that opens up and has y -intercept 8.

ANSWER:

$$y = \frac{4}{3}(x+3)^2 - 4$$