Mid-Chapter Quiz: Lessons 13-1 through 13-3

Simplify each expression.

1. $\cot\theta\sec\theta$

ANSWER:

$\csc\theta$

2. $\dfrac{1-\cos^2\theta}{\sin^2\theta}$

ANSWER:

1

3. $\dfrac{1}{\cos\theta}-\dfrac{\sin^2\theta}{\cos\theta}$

ANSWER:

$\cos\theta$

4. $\cos\left(\dfrac{\pi}{2}-\theta\right)\csc\theta$

ANSWER:

1

5. **HISTORY** In 1861, the United States 34-star flag was adopted. For this flag, $\tan\theta=\dfrac{31.5}{51}$. Find sin θ.

ANSWER:

$\dfrac{31.5\sqrt{3593.25}}{3593.25}$

Find the value of each expression.

6. $\sin\theta$, if $\cos\theta=\dfrac{3}{5};0°<\theta<90°$

ANSWER:

$\dfrac{4}{5}$

7. $\csc\theta$, if $\cot\theta=\dfrac{1}{2};270°<\theta<360°$

ANSWER:

$-\dfrac{\sqrt{5}}{2}$

8. $\tan\theta$, if $\sec\theta=\dfrac{4}{3};0°<\theta<90°$

ANSWER:

$\dfrac{\sqrt{7}}{3}$

9. **MULTIPLE CHOICE** Which of the following is equivalent to $\dfrac{\cos\theta}{1-\sin^2\theta}$?

A. $\cos\theta$
B. $\csc\theta$
C. $\tan\theta$
D. $\sec\theta$

ANSWER:

D

10. **AMUSEMENT PARKS** Suppose a child on a merry-go-round is seated on an outside horse. The diameter of the merry-go-round is 16 meters. The angle of inclination is represented by the equation, $\tan\theta=\dfrac{v^2}{gR'}$ where R is the radius of the circular path, v is the speed in meters per second, and g is 9.8 meters per second squared.
a. If the sine of the angle of inclination of the child is $\dfrac{1}{5}$, what is the angle of inclination made by the child?
b. What is the velocity of the merry-go-round?
c. If the speed of the merry-go-round is 3.6 meters per second, what is the value of the angle of inclination of a rider?

ANSWER:

a. about 11.5°
b. about 4 m/s
c. about 9.4°

Mid-Chapter Quiz: Lessons 13-1 through 13-3

Verify that each of the following is an identity.

11. $\cot^2\theta + 1 = \dfrac{\cot\theta}{\cos\theta \cdot \sin\theta}$

ANSWER:

$\cot^2\theta + 1 \stackrel{?}{=} \dfrac{\cot\theta}{\cos\theta \cdot \sin\theta}$

$\csc^2\theta \stackrel{?}{=} \dfrac{\cos\theta}{\cos\theta \cdot \sin\theta \cdot \sin\theta}$

$\csc^2\theta \stackrel{?}{=} \dfrac{1}{\sin^2\theta}$

$\csc^2\theta = \csc^2\theta$ ✓

12. $\dfrac{\cos\theta\csc\theta}{\cot\theta} = 1$

ANSWER:

$\dfrac{\cos\theta\csc\theta}{\cot\theta} \stackrel{?}{=} 1$

$\dfrac{\cos\theta}{\sin\theta} \cdot \tan\theta \stackrel{?}{=} 1$

$\dfrac{\cos\theta}{\sin\theta} \cdot \dfrac{\sin\theta}{\cos\theta} \stackrel{?}{=} 1$

$1 = 1$ √

13. $\dfrac{\sin\theta\tan\theta}{1-\cos\theta} = (1+\cos\theta)\sec\theta$

ANSWER:

$\dfrac{\sin\theta\tan\theta}{1-\cos\theta} \stackrel{?}{=} (1+\cos\theta)\sec\theta$

$\dfrac{\sin\theta\left(\frac{\sin\theta}{\cos\theta}\right)}{1-\cos\theta} \stackrel{?}{=} \left(1+\cos\theta\right)\dfrac{1}{\cos\theta}$

$\dfrac{\frac{\sin^2\theta}{\cos\theta}}{1-\cos\theta} \stackrel{?}{=} \dfrac{1}{\cos\theta} + 1$

$\dfrac{\sin^2\theta}{\cos\theta} \cdot \dfrac{1}{1-\cos\theta} \stackrel{?}{=} \dfrac{1}{\cos\theta} + 1$

$\dfrac{\sin^2\theta}{\cos\theta(1-\cos\theta)} \stackrel{?}{=} \dfrac{1}{\cos\theta} + 1$

$\dfrac{1-\cos^2\theta}{\cos\theta(1-\cos\theta)} \stackrel{?}{=} \dfrac{1}{\cos\theta} + 1$

$\dfrac{(1-\cos\theta)(1+\cos\theta)}{\cos\theta(1-\cos\theta)} \stackrel{?}{=} \dfrac{1}{\cos\theta} + 1$

$\dfrac{1+\cos\theta}{\cos\theta} \stackrel{?}{=} \dfrac{1}{\cos\theta} + 1$

$\dfrac{1}{\cos\theta} + 1 = \dfrac{1}{\cos\theta} + 1$ √

14. $\tan\theta(1-\sin\theta)=\dfrac{\cos\theta\sin\theta}{1+\sin\theta}$

ANSWER:

$\tan\theta(1-\sin\theta)\stackrel{?}{=}\dfrac{\cos\theta\sin\theta}{1+\sin\theta}$

$\tan\theta(1-\sin\theta)\stackrel{?}{=}\dfrac{\cos\theta\sin\theta}{1+\sin\theta}\cdot\dfrac{1-\sin\theta}{1-\sin\theta}$

$\tan\theta(1-\sin\theta)\stackrel{?}{=}\dfrac{\cos\theta\sin\theta(1-\sin\theta)}{1-\sin^2\theta}$

$\tan\theta(1-\sin\theta)\stackrel{?}{=}\dfrac{\cos\theta\sin\theta(1-\sin\theta)}{\cos^2\theta}$

$\tan\theta(1-\sin\theta)\stackrel{?}{=}\dfrac{\sin\theta(1-\sin\theta)}{\cos\theta}$

$\tan\theta(1-\sin\theta)\stackrel{?}{=}\dfrac{\sin\theta}{\cos\theta}\cdot(1-\sin\theta)$

$\tan\theta(1-\sin\theta)=\tan\theta(1-\sin\theta)$ √

15. **COMPUTER** The front of a computer monitor is usually measured along the diagonal of the screen as shown below.

a. Find *h*.

b. Using the diagram shown, show that $\cot\theta=\dfrac{\cos\theta}{\sin\theta}$.

ANSWER:

a. $15^2=12^2+h^2;225=144+h^2;81=h^2;h=9$

b. $\cot\theta=\frac{12}{9};\dfrac{\cos\theta}{\sin\theta}=\dfrac{\frac{12}{15}}{\frac{9}{15}}=\frac{12}{9}$, so $\frac{12}{9}=\frac{12}{9}$

Verify that each of the following is an identity.

16. $\tan^2\theta+1\stackrel{?}{=}\dfrac{\tan\theta}{\cos\theta\cdot\sin\theta}$

ANSWER:

$\tan^2\theta+1\stackrel{?}{=}\dfrac{\tan\theta}{\cos\theta\cdot\sin\theta}$

$\sec^2\theta\stackrel{?}{=}\dfrac{\tan\theta}{\cos\theta\cdot\sin\theta}$

$\sec^2\theta\stackrel{?}{=}\dfrac{\frac{\sin\theta}{\cos\theta}}{\cos\theta\cdot\sin\theta}$

$\sec^2\theta\stackrel{?}{=}\dfrac{\sin\theta}{\cos^2\theta\cdot\sin\theta}$

$\sec^2\theta\stackrel{?}{=}\dfrac{1}{\cos^2\theta}$

$\sec^2\theta=\sec^2\theta$ ✓

17. $\dfrac{\sin\theta\cdot\sec\theta}{\sec\theta-1}\stackrel{?}{=}(\sec\theta+1)\cot\theta$

ANSWER:

$\dfrac{\sin\theta\cdot\sec\theta}{\sec\theta-1}\stackrel{?}{=}(\sec\theta+1)\cot\theta$

$\dfrac{\sec\theta+1}{\sec\theta+1}\cdot\dfrac{\sin\theta\cdot\sec\theta}{\sec\theta-1}\stackrel{?}{=}(\sec\theta+1)\cot\theta$

$\dfrac{\sin\theta\cdot\sec\theta(\sec\theta+1)}{\sec^2\theta-1}\stackrel{?}{=}(\sec\theta+1)\cot\theta$

$\dfrac{\sin\theta\cdot\frac{1}{\cos\theta}(\sec\theta+1)}{\tan^2\theta}\stackrel{?}{=}(\sec\theta+1)\cot\theta$

$\dfrac{\frac{\sin\theta}{\cos\theta}(\sec\theta+1)}{\tan^2\theta}\stackrel{?}{=}(\sec\theta+1)\cot\theta$

$\dfrac{\tan\theta(\sec\theta+1)}{\tan^2\theta}\stackrel{?}{=}(\sec\theta+1)\cot\theta$

$\dfrac{(\sec\theta+1)}{\tan\theta}\stackrel{?}{=}(\sec\theta+1)\cot\theta$

$(\sec\theta+1)\dfrac{1}{\tan\theta}\stackrel{?}{=}(\sec\theta+1)\cot\theta$

$(\sec\theta+1)\cot\theta=(\sec\theta+1)\cot\theta$ ✓

18. $\sin^2\theta \cdot \tan^2\theta \stackrel{?}{=} \tan^2\theta - \sin^2\theta$

ANSWER:

$$\sin^2\theta \cdot \tan^2\theta \stackrel{?}{=} \tan^2\theta - \sin^2\theta$$
$$\sin^2\theta \cdot \tan^2\theta \stackrel{?}{=} \frac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta$$
$$\sin^2\theta \cdot \tan^2\theta \stackrel{?}{=} \frac{\sin^2\theta - \sin^2\theta\cos^2\theta}{\cos^2\theta}$$
$$\sin^2\theta \cdot \tan^2\theta \stackrel{?}{=} \frac{\sin^2\theta(1-\cos^2\theta)}{\cos^2\theta}$$
$$\sin^2\theta \cdot \tan^2\theta \stackrel{?}{=} \frac{\sin^2\theta(\sin^2\theta)}{\cos^2\theta}$$
$$\sin^2\theta \cdot \tan^2\theta \stackrel{?}{=} \sin^2\theta\frac{\sin^2\theta}{\cos^2\theta}$$
$$\sin^2\theta \cdot \tan^2\theta = \sin^2\theta\tan^2\theta \checkmark$$

19. $\cot\theta(1-\cos\theta) \stackrel{?}{=} \frac{\cos\theta \cdot \sin\theta}{1+\cos\theta}$

ANSWER:

$$\cot\theta(1-\cos\theta) \stackrel{?}{=} \frac{\cos\theta \cdot \sin\theta}{1+\cos\theta}$$
$$\cot\theta(1-\cos\theta) \stackrel{?}{=} \frac{\cos\theta \cdot \sin\theta}{1+\cos\theta} \cdot \frac{1-\cos\theta}{1-\cos\theta}$$
$$\cot\theta(1-\cos\theta) \stackrel{?}{=} \frac{\cos\theta \cdot \sin\theta(1-\cos\theta)}{1-\cos^2\theta}$$
$$\cot\theta(1-\cos\theta) \stackrel{?}{=} \frac{\cos\theta \cdot \sin\theta(1-\cos\theta)}{\sin^2\theta}$$
$$\cot\theta(1-\cos\theta) \stackrel{?}{=} \frac{\cos\theta \cdot (1-\cos\theta)}{\sin\theta}$$
$$\cot\theta(1-\cos\theta) \stackrel{?}{=} \frac{\cos\theta}{\sin\theta}(1-\cos\theta)$$
$$\cot\theta(1-\cos\theta) = \cot\theta(1-\cos\theta) \checkmark$$

Find the exact value of each expression.

20. cos 105°

ANSWER:

$$\frac{\sqrt{2}-\sqrt{6}}{4}$$

21. sin (– 135°)

ANSWER:

$$-\frac{\sqrt{2}}{2}$$

22. tan 15°

ANSWER:

$$2-\sqrt{3}$$

23. cot 75°

ANSWER:

$$2-\sqrt{3}$$

24. **MULTIPLE CHOICE** What is the exact value of $\cos\frac{5\pi}{12}$?

F $\sqrt{2}$

G $\frac{\sqrt{6}+\sqrt{2}}{2}$

H $\frac{\sqrt{6}-\sqrt{2}}{4}$

J $\frac{\sqrt{6}+\sqrt{2}}{4}$

ANSWER:

H

25. Verify that cos 30° cos θ + sin 30° sin θ = sin 60° cos θ + cos 60° sin θ is an identity

ANSWER:

$$\cos 30° \cdot \cos\theta + \sin 30° \cdot \sin\theta \stackrel{?}{=} \sin 60° \cdot \cos\theta + \cos 60° \cdot \sin\theta$$
$$\frac{\sqrt{3}}{2}\cos\theta + \frac{1}{2}\sin\theta = \frac{\sqrt{3}}{2}\cos\theta + \frac{1}{2}\sin\theta \checkmark$$