State whether each sentence is *true* or *false*. If *false*, replace the underlined term to make a true sentence.

1. An infinite geometric series that has a sum is called a convergent series.

ANSWER:
true

2. Mathematical induction is the process of repeatedly composing a function with itself.

ANSWER:
false, iteration

3. The arithmetic means of a sequence are the terms between any two non-successive terms of an arithmetic sequence.

ANSWER:
true

4. A term is a list of numbers in a particular order.

ANSWER:
false, sequence

5. The sum of the first n terms of a series is called the partial sum.

ANSWER:
true

6. The formula $a_n = a_{n-2} + a_{n-1}$ is a recursive formula.

ANSWER:
true

7. A geometric sequence is a sequence in which every term is determined by adding a constant value to the previous term.

ANSWER:
false, arithmetic sequence

8. An infinite geometric series that does not have a sum is called a partial sum.

ANSWER:
false, divergent series

9. Eleven and 17 are two geometric means between 5 and 23 in the sequence 5, 11, 17, 23.

ANSWER:
false, arithmetic means

10. Using the Binomial Theorem, $(x - 2)^4$ can be expanded to $x^4 - 8x^3 + 24x^2 - 32x + 16$.

ANSWER:
true

Find the indicated term of each arithmetic sequence.

11. $a_1 = 9$, $d = 3$, $n = 14$

ANSWER:
48

12. $a_1 = -3$, $d = 6$, $n = 22$

ANSWER:
123

13. $a_1 = 10$, $d = –4$, $n = 9$

ANSWER:
–22

14. $a_1 = –1$, $d = –5$, $n = 18$

ANSWER:
–86

Find the arithmetic means in each sequence.

15. –12, __, __, __, 8

ANSWER:
–7, –2, 3

16. 15, __, __, 29

ANSWER:
$\frac{59}{3}, \frac{73}{3}$

17. 12, __, __, __, __, –8

ANSWER:
8, 4, 0, –4

18. 72, __, __, __, 24

ANSWER:
60, 48, 36

19. **BANKING** Carson saves $40 every 2 months. If he saves at this rate for two years, how much will he have at the end of two years?

ANSWER:
$480

Find S_n for each arithmetic series.

20. $a_1 = 16$, $a_n = 48$, $n = 6$

ANSWER:
192

21. $a_1 = 8$, $a_n = 96$, $n = 20$

ANSWER:
1040

22. 9 + 14 + 19 + … + 74

ANSWER:
581

23. 16 + 7 + -2 + … + –65

ANSWER:
–245

24. **DRAMA** Laura has a drama performance in 12 days. She plans to practice her lines each night. On the first night she rehearses her lines 2 times. The next night she rehearses her lines 4 times. The third night she rehearses her lines 6 times. On the eleventh night, how many times has she rehearsed her lines?

ANSWER:

132

Find the sum of each arithmetic series.

25. $\sum_{k=5}^{21}(3k-2)$

ANSWER:

629

26. $\sum_{k=0}^{10}(6k-1)$

ANSWER:

319

27. $\sum_{k=4}^{12}(-2k+5)$

ANSWER:

–99

Find the indicated term for each geometric sequence.

28. $a_1 = 5$, $r = 2$, $n = 7$

ANSWER:

320

29. $a_1 = 11$, $r = 3$, $n = 3$

ANSWER:

99

30. $a_1 = 128, r = -\frac{1}{2}, n = 5$

ANSWER:

8

31. a_8 for $\frac{1}{8}, \frac{3}{8}, \frac{9}{8}, \ldots$

ANSWER:

$\frac{2187}{8}$

Find the geometric means in each sequence.

32. 6, __, __, 162

ANSWER:

18, 54

33. 8, __, __, __, 648

ANSWER:

$\pm 24, 72, \pm 216$

34. –4, __, __, 108

ANSWER:

12, –36

35. **SAVINGS** Nolan has a savings account with a current balance of $1500. What would be Nolan's account balance after 4 years if he receives 5% interest annually?

ANSWER:

$1823.26

Find S_n for each geometric series.

36. $a_1 = 15$, $r = 2$, $n = 4$

ANSWER:

225

37. $a_1 = 9$, $r = 4$, $n = 6$

ANSWER:

12,285

38. 5 – 10 + 20 – … to 7 terms

ANSWER:

215

39. 243 + 81 + 27 + … to 5 terms

ANSWER:

363

Evaluate the sum of each geometric series.

40. $\sum_{k=1}^{7} 3 \cdot (-2)^{n-1}$

ANSWER:

129

41. $\sum_{k=1}^{8} -1\left(\frac{2}{3}\right)^{k-1}$

ANSWER:

$-\frac{6305}{2187}$

42. **ADVERTISING** Natalie is handing out fliers to advertise the next student council meeting. She hands out fliers to 4 people. Then, each of those 4 people hand out 4 fliers to 4 other people. Those 4 then hand out 4 fliers to 4 new people. If Natalie is considered the first round, how many people will have been given fliers after 4 rounds?

ANSWER:

85

Find the sum of each infinite series, if it exists.

43. $a_1 = 8, r = \frac{3}{4}$

ANSWER:

32

44. $\frac{5}{6}-\frac{20}{18}+\frac{80}{54}-\frac{320}{162}+\ldots$

ANSWER:
does not exist

45. $\sum_{k=1}^{\infty} 3\left(\frac{1}{2}\right)^{k-1}$

ANSWER:
6

46. **PHYSICAL SCIENCE** Maddy drops a ball off of a building that is 60 feet high. Each time the ball bounces, it bounces back to $\frac{2}{3}$ its previous height. If the ball continues to follow this pattern, what will be the total distance that the ball travels?

ANSWER:
300 ft

Find the first five terms of each sequence.

47. $a_1 = -3$, $a_{n+1} = a_n + 4$

ANSWER:
–3, 1, 5, 9, 13

48. $a_1 = 5$, $a_{n+1} = 2a_n - 5$

ANSWER:
5, 5, 5, 5, 5

49. $a_1 = 1$, $a_{n+1} = a_n + 5$

ANSWER:
1, 6, 11, 16, 21

50. **SAVINGS** Sari has a savings account with a $12,000 balance. She has a 5% interest rate that is compounded monthly. Every month Sari adds $500 to the account. The recursive formula $b_n = 1.05b_{n-1} + 500$ describes the balance in Sari's savings account after n months. Find the balance of Sari's account after 3 months. Round your answer to the nearest penny.

ANSWER:
$15,467.75

Find the first three iterates of each function for the given initial value.

51. $f(x) = 2x + 1$, $x_0 = 3$

ANSWER:
7, 15, 31

52. $f(x) = 5x - 4$, $x_0 = 1$

ANSWER:
1, 1, 1

53. $f(x) = 6x - 1$, $x_0 = 2$

ANSWER:
11, 65, 389

54. $f(x) = 3x + 1$, $x_0 = 4$

ANSWER:

13, 40, 121

Expand each binomial.

55. $(a + b)^3$

ANSWER:

$a^3 + 3a^2b + 3ab^2 + b^3$

56. $(y - 3)^7$

ANSWER:

$y^7 - 21y^6 + 189y^5 - 945y^4 + 2835y^3 - 5103y^2 + 5103y - 2187$

57. $(3 - 2z)^5$

ANSWER:

$-32z^5 + 240z^4 - 720z^3 + 1080z^2 - 810z + 243$

58. $(4a - 3b)^4$

ANSWER:

$256a^4 - 768a^3b + 864a^2b^2 - 432ab^3 + 81b^4$

59. $\left(x - \frac{1}{4}\right)^5$

ANSWER:

$x^5 - \frac{5}{4}x^4 + \frac{5}{8}x^3 - \frac{5}{32}x^2 + \frac{5}{256}x - \frac{1}{1024}$

Find the indicated term of each expression.

60. third term of $(a + 2b)^8$

ANSWER:

$112a^6b^2$

61. sixth term of $(3x + 4y)^7$

ANSWER:

$193{,}536x^2y^5$

62. second term of $(4x - 5)^{10}$

ANSWER:

$-13{,}107{,}200x^9$

Prove that each statement is true for all positive integers.

63. $2+6+12+\ldots+n(n+1)=\dfrac{n(n+1)(n+2)}{3}$

ANSWER:

Step 1 When n = 1, the left side of the equation is equal to 2. The right side of the equation is also equal to 2. So the equation is true for n = 1.

Step 2 Assume that

$2+6+12+\ldots+k(k+1)=\dfrac{k(k+1)(k+2)}{3}$ for some positive integer k

Step 3: $1\cdot 2+2\cdot 3+3\cdot 42+\ldots+k(k+1)+(k+1)(k+2)$

$$=\frac{k(k+1)(k+2)}{3}+(k+1)(k+2)$$
$$=\frac{k(k+1)(k+2)}{3}+\frac{3(k+1)(k+2)}{3}$$
$$=\frac{(k+1)\left[k(k+2)+3(k+2)\right]}{3}$$
$$=\frac{(k+1)(k+2)(k+3)}{3}$$
$$=\frac{(k+1)((k+1)+1)((k+1)+2)}{3}$$

The last expression is the right side of the equation to be proved, where $n = k + 1$. Thus, the equation is true for $n = k + 1$. Therefore,

$2+6+12+\ldots+n(n+1)=\dfrac{n(n+1)(n+2)}{3}$ for all positive integers n.

64. $7^n - 1$ is divisible by 6.

ANSWER:

Step 1: When $n = 1$, $7^1 - 1 = 7 - 1$ or 6. Since 6 divided by 6 is 1, the statement is true for n = 1.

Step 2: Assume that $7^k - 1$ is divisible by 6 for some positive integer k. This means that $7^k - 1 = 6r$ for some whole number r.

Step 3:

$$\begin{aligned}7^k-1&=6r\\7^k&=6r+1\\7^{k+1}&=42r+7\\7^{k+1}-1&=42r+7-1\\7^{k+1}-1&=42r+6\\7^{k+1}-1&=6(7r+1)\end{aligned}$$

Since r is a whole number, $7r + 1$ is a whole number. Thus, $7^{k+1} - 1$ is divisible by 6, so the statement is true for $n = k + 1$. Therefore, $7^n - 1$ is divisible by 6 for all positive integers n.

65. $5^n - 1$ is divisible by 4.

ANSWER:

Step 1: When $n = 1$, $5^1 - 1 = 5 - 1$ or 4. Since 4 divided by 4 is 1, the statement is true for $n = 1$.

Step 2: Assume that $5^k - 1$ is divisible by 4 for some positive integer k. This means that $5^k - 1 = 4r$ for some whole number r.

Step 3:

$$\begin{aligned} 5^k - 1 &= 4r \\ 5^k &= 4r + 1 \\ 5^{k+1} &= 20r + 5 \\ 5^{k+1} - 1 &= 20r + 5 - 1 \\ 5^{k+1} - 1 &= 20r + 4 \\ 5^{k+1} - 1 &= 4(5r + 1) \end{aligned}$$

Since r is a whole number, $5r + 1$ is a whole number. Thus, $5^{k+1} - 1$ is divisible by 4, so the statement is true for $n = k + 1$. Therefore, $5^n - 1$ is divisible by 4 for all positive integers n.

Find a counterexample for each statement.

66. $8n + 3$ is divisible by 11.

ANSWER:

$n = 2$

67. $6^{n+1} - 2$ is divisible by 17.

ANSWER:

$n = 2$

68. $n^2 + 2n + 4$ is prime.

ANSWER:

$n = 2$

69. $n + 19$ is prime.

ANSWER:

$n = 1$