Choose the correct term to complete each sentence.

1. The ____________ can be used to find the sine or cosine of 75° if the sine and cosine of 90° and 15° are known.

ANSWER:
difference of angles identity

2. The identities $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\cot\theta = \frac{\cos\theta}{\sin\theta}$ are examples of ____________.

ANSWER:
quotient identities

3. A ____________ is an equation involving trigonometric functions that is true for all values for which every expression in the equation is defined.

ANSWER:
trigonometric identity

4. The ________ can be used to find sin 60° using 30° as a reference.

ANSWER:
double-angle identity

5. A _________ is true for only certain values of the variable.

ANSWER:
trigonometric equation

6. The _________ formula can be used to find $\cos 22\frac{1}{2}°$.

ANSWER:
half-angle

7. The identities $\csc\theta = \frac{1}{\sin\theta}$ and $\sec\theta = \frac{1}{\cos\theta}$ are examples of __________.

ANSWER:
reciprocal identities

8. The ________ can be used to find the sine or cosine of 120° if the sine and cosine 90° and 30° are known.

ANSWER:
sum of angles identity

9. $\cos^2\theta + \sin^2\theta = 1$ is an example of a ________.

ANSWER:
Pythagorean identity

Find the value of each expression.

10. $\sin\theta$, if $\cos\theta = \frac{\sqrt{2}}{2}$ and $270° < \theta < 360°$

ANSWER:
$-\frac{\sqrt{2}}{2}$

11. $\sec\theta$, if $\cot\theta = \frac{\sqrt{2}}{2}$ and $90° < \theta < 180°$

ANSWER:
$-\sqrt{3}$

12. $\tan\theta$, if $\cot\theta = 2$ and $0° < \theta < 90°$

ANSWER:
$\frac{1}{2}$

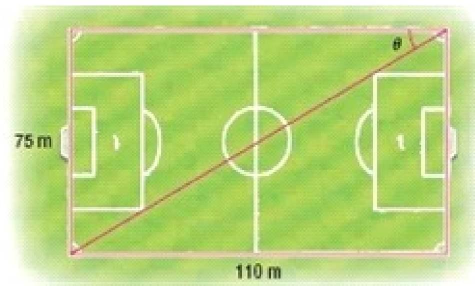
13. $\cos\theta$, if $\sin\theta = -\frac{3}{5}$ and $180° < \theta < 270°$

ANSWER:
$-\frac{4}{5}$

14. $\csc\theta$, if $\cot\theta = -\frac{4}{5}$ and $270° < \theta < 360°$

ANSWER:
$-\frac{\sqrt{41}}{5}$

15. **SOCCER** For international matches, the maximum dimensions of a soccer field are 110 meters by 75 meters. Find sin θ.



ANSWER:
First find the length of the diagonal:

$$\begin{aligned} 75^2 + 110^2 &= c^2; \\ 5625 + 12{,}100 &= c^2; \\ 17{,}725 &= c^2; \\ c &= 5\sqrt{709}; \\ \sin\theta &= \frac{75}{5\sqrt{709}} \\ &= \frac{15\sqrt{709}}{709} \end{aligned}$$

Simplify each expression.

16. $1 - \tan\theta\sin\theta\cos\theta$

ANSWER:
$\cos^2\theta$

17. $\tan\theta\csc\theta$

ANSWER:
$\sec\theta$

18. $\sin\theta + \cos\theta\cot\theta$

ANSWER:

$\csc\theta$

19. $\cos\theta(1+\tan^2\theta)$

ANSWER:

$\sec\theta$

Verify that each of the following is an identity.

20. $\tan\theta\cos\theta + \cot\theta\sin\theta = \sin\theta + \cos\theta$

ANSWER:

$$\tan\theta\cos\theta + \cot\theta\sin\theta \stackrel{?}{=} \sin\theta + \cos\theta$$
$$\frac{\sin\theta}{\cos\theta}\cdot\cos\theta + \frac{\cos\theta}{\sin\theta}\cdot\sin\theta \stackrel{?}{=} \sin\theta + \cos\theta$$
$$\sin\theta + \cos\theta = \sin\theta + \cos\theta\ \checkmark$$

21. $\dfrac{\cos\theta}{\cot\theta} + \dfrac{\sin\theta}{\tan\theta} = \sin\theta + \cos\theta$

ANSWER:

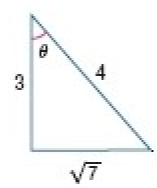
$$\frac{\cos\theta}{\cot\theta} + \frac{\sin\theta}{\tan\theta} \stackrel{?}{=} \sin\theta + \cos\theta$$
$$\cos\theta \div \frac{\cos\theta}{\sin\theta} + \sin\theta \div \frac{\sin\theta}{\cos\theta} \stackrel{?}{=} \sin\theta + \cos\theta$$
$$\cos\theta\cdot\frac{\sin\theta}{\cos\theta} + \sin\theta\cdot\frac{\cos\theta}{\sin\theta} \stackrel{?}{=} \sin\theta + \cos\theta$$
$$\sin\theta + \cos\theta = \sin\theta + \cos\theta\checkmark$$

22. $\sec^2\theta - 1 = \dfrac{\sin^2\theta}{1-\sin^2\theta}$

ANSWER:

$$\sec^2\theta - 1 \stackrel{?}{=} \frac{\sin^2\theta}{1-\sin^2\theta}$$
$$\sec^2\theta - 1 \stackrel{?}{=} \frac{\sin^2\theta}{\cos^2\theta}$$
$$\sec^2\theta - 1 \stackrel{?}{=} \tan^2\theta$$
$$\sec^2\theta - 1 = \sec^2\theta - 1\checkmark$$

23. **GEOMETRY** The right triangle shown is used in a special quilt. Use the measures of the sides of the triangle to show that $\tan^2\theta + 1 = \sec^2\theta$.



ANSWER:

$$\tan^2\theta + 1 = \left(\frac{\sqrt{7}}{3}\right)^2 + 1 = \frac{7}{9} + 1$$
$$= \frac{7}{9} + \frac{9}{9} = \frac{16}{9};$$
$$\sec^2\theta = \left(\frac{4}{3}\right)^2 = \frac{16}{9}$$

Find the exact value of each expression.

24. $\cos(-135°)$

ANSWER:

$-\dfrac{\sqrt{2}}{2}$

25. $\cos 15°$

ANSWER:

$\frac{\sqrt{6}+\sqrt{2}}{4}$

26. $\sin 210°$

ANSWER:

$-\frac{1}{2}$

27. $\sin 105°$

ANSWER:

$\frac{\sqrt{6}+\sqrt{2}}{4}$

28. $\tan 75°$

ANSWER:

$\sqrt{3}+2$

29. $\cos 105°$

ANSWER:

$\frac{-\sqrt{6}+\sqrt{2}}{4}$

Verify that each of the following is an identity.

30. $\sin(\theta+90)=\cos\theta$

ANSWER:

$$\begin{aligned}\sin(\theta+90) &\stackrel{?}{=} \cos\theta\\ \sin\theta\cos 90° + \cos\theta\sin 90° &\stackrel{?}{=} \cos\theta\\ \sin\theta(0)+\cos\theta(1) &\stackrel{?}{=} \cos\theta\\ \cos\theta &= \cos\theta \checkmark\end{aligned}$$

31. $\sin\left(\frac{3\pi}{2}-\theta\right)=-\cos\theta$

ANSWER:

$$\begin{aligned}\sin\left(\frac{3\pi}{2}-\theta\right) &\stackrel{?}{=} -\cos\theta\\ \sin\frac{3\pi}{2}\cos\theta-\cos\frac{3\pi}{2}\sin\theta &\stackrel{?}{=} -\cos\theta\\ (-1)\cos\theta-(0)\sin\theta &\stackrel{?}{=} -\cos\theta\\ -\cos\theta &= -\cos\theta \checkmark\end{aligned}$$

32. $\tan(\theta-\pi)=\tan\theta$

ANSWER:

$$\begin{aligned}\tan(\theta-\pi) &\stackrel{?}{=} \tan\theta\\ \frac{\tan\theta-\tan\pi}{1+\tan\theta\tan\pi} &\stackrel{?}{=} \tan\theta\\ \frac{\tan\theta-0}{1+\tan\theta(0)} &\stackrel{?}{=} \tan\theta\\ \frac{\tan\theta}{1} &\stackrel{?}{=} \tan\theta\\ \tan\theta &= \tan\theta \checkmark\end{aligned}$$

Find the exact values of $\sin 2\theta, \cos 2\theta, \sin\frac{\theta}{2}$, and $\cos\frac{\theta}{2}$ for each of the following.

33. $\cos\theta = \frac{4}{5}; 0° < \theta < 90°$

ANSWER:

$\sin 2\theta = \frac{24}{25}, \cos 2\theta = \frac{7}{25}, \sin\frac{\theta}{2} = \frac{\sqrt{10}}{10}$, and $\cos\frac{\theta}{2} = \frac{3\sqrt{10}}{10}$

34. $\sin\theta = -\frac{1}{4}; 180° < \theta < 270°$

ANSWER:

$\sin 2\theta = \frac{\sqrt{15}}{8}, \cos 2\theta = \frac{7}{8}, \sin\frac{\theta}{2} = \frac{\sqrt{2}\sqrt{4+\sqrt{15}}}{4}$, and $\cos\frac{\theta}{2} = -\frac{\sqrt{2}\sqrt{4-\sqrt{15}}}{4}$

35. $\cos\theta = -\frac{2}{3}; \frac{\pi}{2} < \theta < \pi$

ANSWER:

$\sin 2\theta = -\frac{4\sqrt{5}}{9}$, $\cos 2\theta = -\frac{1}{9}$, $\sin\frac{\theta}{2} = \frac{\sqrt{30}}{6}$, and $\cos\frac{\theta}{2} = \frac{\sqrt{6}}{6}$

36. **BASEBALL** The infield of a baseball diamond is a square with side length 90 feet.

a. Find the length of the diagonal.

b. Write the ratio for sin 45° using the lengths of the baseball diamond.

c. Use the formula $\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$ to verify the ratio you wrote in part b.

ANSWER:

a. $c^2 = 90^2 + 90^2; c^2 = 8100 + 8100; c^2 = 16,200; c = 90\sqrt{2}$

b. $\sin 45° = \frac{90}{90\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

c.

$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}; \sin\frac{90°}{2} = \pm\sqrt{\frac{1-\cos 90}{2}};$

$\sin\frac{90°}{2} = \pm\sqrt{\frac{1-0}{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

Find all solutions of each equation for the given interval.

37. $2\cos\theta - 1 = 0; 0° \le \theta < 360°$

ANSWER:

60°, 300°

38. $4\cos^2\theta - 1 = 0; 0 \le \theta < 2\pi$

ANSWER:

$\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

39. $\sin 2\theta + \cos\theta = 0; 0° \le \theta < 360°$

ANSWER:
90°, 210°, 270°, 330°

40. $\sin^2\theta = 2\sin\theta + 3; 0° \le \theta < 360°$

ANSWER:
270°

41. $4\cos^2\theta - 4\cos\theta + 1 = 0; 0 \le \theta < 2\pi$

ANSWER:
$\frac{\pi}{3}, \frac{5\pi}{3}$