Choose a word or term from the list above that best completes each statement or phrase.

1. A function of the form $f(x) = b^x$ where $b > 1$ is a(n) ______________ function.

SOLUTION:

A function of the form $f(x) = b^x$ where $b > 1$ is a(n) exponential growth function.

2. In $x = b^y$, the variable y is called the ______________ of x.

SOLUTION:

In $x = b^y$, the variable y is called the _logarithm_ of x.

3. Base 10 logarithms are called ______________.

SOLUTION:

Base 10 logarithms are called _common logarithms_.

4. A(n) ______________ is an equation in which variables occur as exponents.

SOLUTION:

A(n) _exponential equation_ is an equation in which variables occur as exponents.

5. The ______________ allows you to write equivalent logarithmic expressions that have different bases.

SOLUTION:

The _change of base formula_ allows you to write equivalent logarithmic expressions that have different bases.

6. The base of the exponential function, $A(t) = a(1 - r)t$, $1 - r$ is called the ______________.

SOLUTION:

The base of the exponential function, $A(t) = a(1 - r)t$, $1 - r$ is called the _decay factor_.

7. The function $y = \log_b x$, where $b > 0$ and $b \neq 1$, is called a(n) ______________.

SOLUTION:

The function $y = \log_b x$, where $b > 0$ and $b \neq 1$, is called a(n) _logarithmic function_.

8. An exponential function with base e is called the ______________.

SOLUTION:

An exponential function with base e is called the _natural base exponential function_.

9. The logarithm with base e is called the ______________.

SOLUTION:

The logarithm with base e is called the _natural logarithm_.

10. The number e is referred to as the ______________.

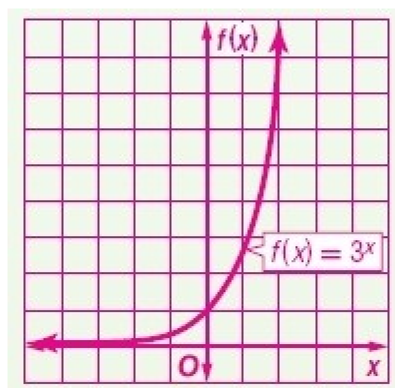
SOLUTION:

The number e is referred to as the _natural base_.

Graph each function. State the domain and range.

11. $f(x) = 3^x$

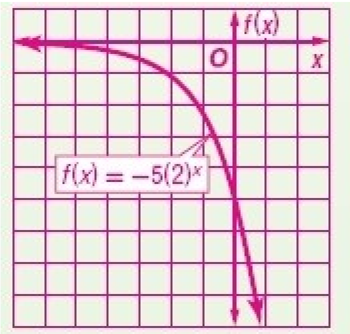
SOLUTION:



The function is defined for all values of x. Therefore, the domain is set of all real numbers.
The value of $f(x)$ tends to ∞ as x tends to ∞.
The value of $f(x)$ tends to 0 as x tends to $-\infty$.
Therefore, the range of the function is $\{f(x) \mid f(x) > 0\}$.

12. $f(x) = -5(2)^x$

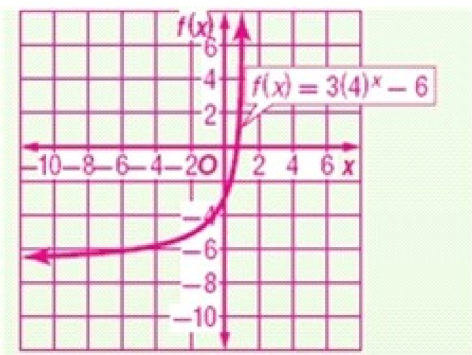
SOLUTION:



The function is defined for all values of x. Therefore, the domain is set of all real numbers.
The value of $f(x)$ tends to ∞ as x tends to ∞.
The value of $f(x)$ tends to 0 as x tends to –∞.
Therefore, the range of the function is $\{f(x) \mid f(x) < 0\}$.

13. $f(x) = 3(4)^x - 6$

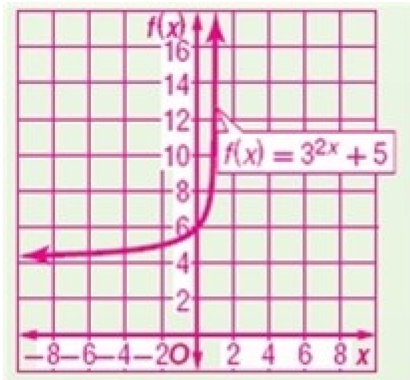
SOLUTION:



The function is defined for all values of x. Therefore, the domain is set of all real numbers.
The value of $f(x)$ tends to ∞ as x tends to ∞.
The value of $f(x)$ tends to –6 as x tends to –∞.
Therefore, the range of the function is $\{f(x) \mid f(x) > -6\}$.

14. $f(x) = 3^{2x} + 5$

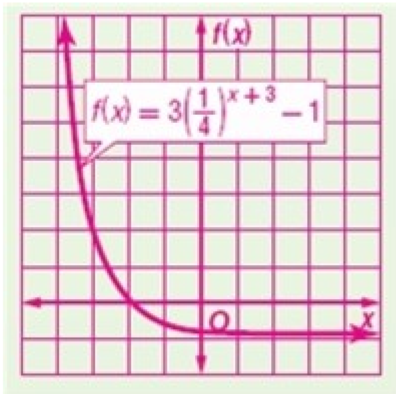
SOLUTION:



The function is defined for all values of x. Therefore, the domain is set of all real numbers.
The value of $f(x)$ tends to ∞ as x tends to ∞.
The value of $f(x)$ tends to 5 as x tends to –∞.
Therefore, the range of the function is $\{f(x) \mid f(x) > 5\}$.

15. $f(x) = 3\left(\frac{1}{4}\right)^{x+3} - 1$

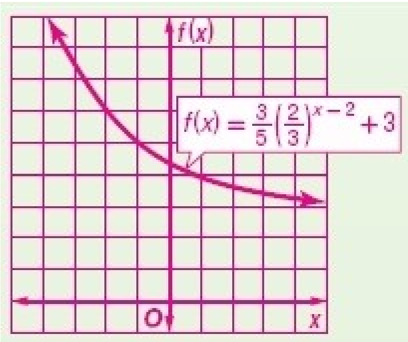
SOLUTION:



The function is defined for all values of x. Therefore, the domain is set of all real numbers.
The value of $f(x)$ tends to ∞ as x tends to –∞.
The value of $f(x)$ tends to –1 as x tends to ∞.
Therefore, the range of the function is $\{f(x) \mid f(x) > -1\}$.

16. $f(x) = \frac{3}{5}\left(\frac{2}{3}\right)^{x-2} + 3$

SOLUTION:



The function is defined for all values of x. Therefore, the domain is set of all real numbers.
The value of $f(x)$ tends to ∞ as x tends to –∞.
The value of $f(x)$ tends to 3 as x tends to ∞.
Therefore, the range of the function is $\{f(x) \mid f(x) > 3\}$.

17. **POPULATION** A city with a population of 120,000 decreases at a rate of 3% annually.

a. Write the function that represents this situation.

b. What will the population be in 10 years?

SOLUTION:

a. Exponential decay with a constant percent increase over specific time periods is modeled by $f(x) = a(1 - r)^x$.
Substitute 120,000 for a and 0.03 for r.

$$f(x) = 120000(1 - 0.03)^x$$
$$f(x) = 120000(0.97)^x$$

b. Substitute 10 for x and evaluate.

$$f(10) = 120000(0.97)^{10} \approx 88491$$

The population will be about 88,491.

Solve each equation or inequality.

18. $16^x = \frac{1}{64}$

SOLUTION:

$$16^x = \frac{1}{64}$$
$$\left(2^4\right)^x = \left(\frac{1}{2}\right)^6$$
$$2^{2x} = 2^{-6}$$

$$\log 2^{4x} = \log 2^{-6}$$
$$4x \log 2 = -6 \log 2$$
$$x = \frac{-6 \log 2}{4 \log 2} = -\frac{3}{2}$$

The solution is $-\frac{3}{2}$.

19. $3^{4x} = 9^{3x+7}$

SOLUTION:

$$3^{4x} = 9^{3x+7}$$
$$3^{4x} = \left(3^2\right)^{3x+7}$$
$$3^{4x} = 3^{6x+14}$$
$$\log 3^{4x} = \log 3^{6x+14}$$
$$4x \log 3 = (6x + 14) \log 3$$
$$4x = 6x + 14$$
$$2x = -14$$
$$x = -7$$

The solution is –7.

20. $64^{3n} = 8^{2n-3}$

SOLUTION:

$$
\begin{aligned}
64^{3n} &= 8^{2n-3} \\
\left(8^2\right)^{3n} &= 8^{2n-3} \\
8^{6n} &= 8^{2n-3} \\
\log 8^{6n} &= \log 8^{2n-3} \\
6n \log 8 &= (2n-3)\log 8 \\
6n &= 2n-3 \\
4n &= -3 \\
n &= -\frac{3}{4}
\end{aligned}
$$

The solution is $-\frac{3}{4}$.

21. $8^{3-3y} = 256^{4y}$

SOLUTION:

$$
\begin{aligned}
8^{3-3y} &= 256^{4y} \\
\left(2^3\right)^{3-3y} &= \left(2^8\right)^{4y} \\
2^{9-9y} &= 2^{32y} \\
\log 2^{9-9y} &= \log 2^{32y} \\
(9-9y)\log 2 &= 32y \log 2 \\
9-9y &= 32y \\
9 &= 41y \\
y &= \frac{9}{41}
\end{aligned}
$$

The solution is $\frac{9}{41}$.

22. $9^{x-2} > \left(\frac{1}{81}\right)^{x+2}$

SOLUTION:

$$
\begin{aligned}
9^{x-2} &> \left(\frac{1}{81}\right)^{x+2} \\
9^{x-2} &> \left(\frac{1}{9^2}\right)^{x+2} \\
9^{x-2} &> \frac{1}{9^{2x+4}} \\
9^{x-2} &> 9^{-2x-4} \\
\log 9^{x-2} &> \log 9^{-2x-4} \\
(x-2)\log 9 &> (-2x-4)\log 9 \\
x-2 &> -2x-4 \\
3x &> -2 \\
x &> -\frac{2}{3}
\end{aligned}
$$

The solution is $\left\{x \mid x > -\frac{2}{3}\right\}$.

23. $27^{3x} \le 9^{2x-1}$

SOLUTION:

$$
\begin{aligned}
27^{3x} &\le 9^{2x-1} \\
\left(3^3\right)^{3x} &\le \left(3^2\right)^{2x-1} \\
3^{9x} &\le 3^{4x-2} \\
\log 3^{9x} &\le \log 3^{4x-2} \\
9x \log 3 &\le (4x-2)\log 3 \\
9x &\le 4x-2 \\
5x &\le -2 \\
x &\le -\frac{2}{5}
\end{aligned}
$$

The solution is $\left\{x \mid x \le -\frac{2}{5}\right\}$.

24. **BACTERIA** A bacteria population started with 5000 bacteria. After 8 hours there were 28,000 in the sample.
a. Write an exponential function that could be used to model the number of bacteria after x hours if the number of bacteria changes at the same rate.
b. How many bacteria can be expected in the sample after 32 hours?

SOLUTION:

a. At the beginning of the experiment, the time is 0 hours and there are 5000 bacteria cells. Thus, the y-intercept, and the value of a, is 5000. When $x = 8$, the number of bacteria cells is 28,000. Substitute these values into an exponential function to determine the value of b.

$$\begin{aligned} y &= ab^x \\ 28000 &= 5000b^8 \\ 5.6 &= b^8 \\ b &= \sqrt[8]{5.6} \\ &\approx 1.240 \end{aligned}$$

An equation that models the number of bacteria is $y = 5000(1.240)^x$.

b. Substitute 32 for x and evaluate.

$$\begin{aligned} y &= 5000(1.240)^{32} \\ &\approx 4880496 \end{aligned}$$

There will be approximately 4,880,496 bacteria cells after 32 hours.

25. Write $\log_2 \frac{1}{16} = -4$ in exponential form.

SOLUTION:

$$\begin{aligned} \log_2 \frac{1}{16} &= -4 \\ \frac{1}{16} &= 2^{-4} \end{aligned}$$

26. Write $10^2 = 100$ in logarithmic form.

SOLUTION:

$$\begin{aligned} 10^2 &= 100 \\ \log_{10} 10^2 &= \log_{10} 100 \\ 2 &= \log_{10} 100 \end{aligned}$$

Evaluate each expression.

27. $\log_4 256$

SOLUTION:

$$\begin{aligned} \log_4 256 &= \log_4 4^4 \\ &= 4 \log_4 4 \\ &= 4 \end{aligned}$$

28. $\log_2 \frac{1}{8}$

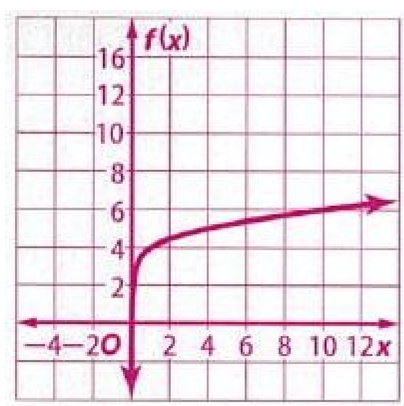
SOLUTION:

$$\begin{aligned} \log_2 \frac{1}{8} &= \log_2 \frac{1}{2^3} \\ &= \log_2 2^{-3} \\ &= -3 \log_2 2 \\ &= -3 \end{aligned}$$

Graph each function.

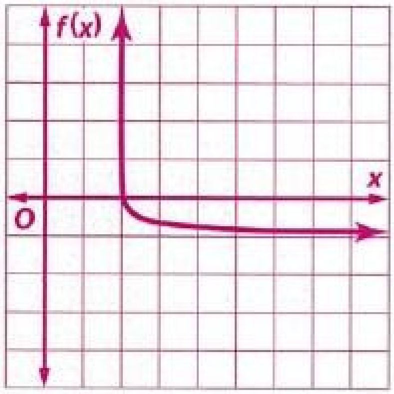
29. $f(x) = 2 \log_{10} x + 4$

SOLUTION:



30. $f(x) = \frac{1}{6}\log_{\frac{1}{3}}(x-2)$

SOLUTION:



Solve each equation or inequality.

31. $\log_4 x = \frac{3}{2}$

SOLUTION:

$$\log_4 x = \frac{3}{2}$$
$$x = 4^{\frac{3}{2}}$$
$$x = \left(\sqrt{4}\right)^3$$
$$x = 8$$

The solution is 8.

32. $\log_2 \frac{1}{64} = x$

SOLUTION:

$$\log_2 \frac{1}{64} = x$$
$$\log_2 \frac{1}{2^6} = x$$
$$\log_2 2^{-6} = x$$
$$-6\log_2 2 = x$$
$$x = -6$$

The solution is –6.

33. $\log_4 x < 3$

SOLUTION:

If $b > 1$, $x > 0$, and $\log_b x < y$, then $0 < x < b^y$ so

$$\log_4 x < 3$$
$$0 < x < 4^3$$
$$0 < x < 64$$

34. $\log_5 x < -3$

SOLUTION:

If $b > 1$, $x > 0$, and $\log_b x < y$, then $0 < x < b^y$ so

$$\log_5 x < -3$$
$$0 < x < 5^{-3}$$
$$0 < x < \frac{1}{5^3}$$
$$0 < x < \frac{1}{125}$$

35. $\log_9(3x-1) = \log_9(4x)$

SOLUTION:

$$\log_9(3x-1) = \log_9(4x)$$
$$3x - 1 = 4x$$
$$x = -1$$

The value of *x* makes the argument negative. Logarithms are not defined for negative numbers. Therefore, there is no solution.

36. $\log_2(x^2-18)=\log_2(-3x)$

SOLUTION:

$$\begin{aligned}\log_2(x^2-18)&=\log_2(-3x)\\ x^2-18&=-3x\\ x^2+3x-18&=0\\ (x+6)(x-3)&=0\end{aligned}$$

By Zero Product Property:

$$\begin{aligned}&x+6=0\text{or} && x-3=0\\ &x=-6 \quad \text{or} && x=3\end{aligned}$$

The x-value 3 makes the argument negative. Logarithms are not defined for negative numbers. Therefore, the solution is –6.

37. $\log_3(3x+4)\le\log_3(x-2)$

SOLUTION:

$$\begin{aligned}\log_3(3x+4)&\le\log_3(x-2)\\ 3x+4&\le x-2\\ 2x&\le-6\\ x&\le-3\end{aligned}$$

The value of x makes the argument negative. Logarithms are not defined for negative numbers. Therefore, there is no solution.

38. **EARTHQUAKE** The magnitude of an earthquake is measured on a logarithmic scale called the Richter scale. The magnitude M is given by $M = \log_{10} x$, where x represents the amplitude of the seismic wave causing ground motion. How many times as great is the amplitude caused by an earthquake with a Richter scale rating of 10 as an aftershock with a Richter scale rating of 7?

SOLUTION:

Substitute 10 and 7 for M and find the value of x.

$$\begin{aligned}M&=\log_{10}x\\ 10&=\log_{10}x\\ x&=10^{10}\\ 7&=\log_{10}x\\ x&=10^{7}\end{aligned}$$

The ratio between the amplitude is 10^3.

Use $\log_5 16 \approx 1.7227$ and $\log_5 2 \approx 0.4307$ to approximate the value of each expression.

39. $\log_5 8$

SOLUTION:

$$\begin{aligned}\log_5 8&=\log_5\left(\frac{16}{2}\right)\\ &=\log_5 16-\log_5 2\\ &\approx 1.7227-0.4307\\ &\approx 1.2920\end{aligned}$$

40. $\log_5 64$

SOLUTION:

$$\begin{aligned}\log_5 64&=\log_5(16\cdot2\cdot2)\\ &=\log_5 16+\log_5 2+\log_5 2\\ &\approx 1.7227+0.4307+0.4307\\ &\approx 2.5841\end{aligned}$$

41. $\log_5 4$

SOLUTION:

$$\begin{aligned}\log_5 4 &= \log_5 2^2\\ &= 2\log_5 2\\ &\approx 2(0.4307)\\ &= 0.8614\end{aligned}$$

42. $\log_5 \frac{1}{8}$

SOLUTION:

$$\begin{aligned}\log_5 \frac{1}{8} &= \log_5 \frac{1}{2^3}\\ &= \log 2^{-3}\\ &= -3\log_5 2\\ &\approx -3(0.4307)\\ &= -1.2921\end{aligned}$$

43. $\log_5 \frac{1}{2}$

SOLUTION:

$$\begin{aligned}\log_5 \frac{1}{2} &= \log 2^{-1}\\ &= -1\log_5 2\\ &\approx -1(0.4307)\\ &= -0.4307\end{aligned}$$

Solve each equation. Check your solution.

44. $\log_5 x - \log_5 2 = \log_5 15$

SOLUTION:

$$\begin{aligned}\log_5 x - \log_5 2 &= \log_5 15\\ \log_5 \frac{x}{2} &= \log_5 15\\ \frac{x}{2} &= 15\\ x &= 30\end{aligned}$$

Substitute 30 for x and check the solution.

$$\begin{aligned}\log_5 30 - \log_5 2 &= \log_5 15\\ \log_5(15\cdot 2) - \log_5 2 &= \log_5 15\\ \log_5 15 + \log_5 2 - \log_5 2 &= \log_5 15\\ \log_5 15 &= \log_5 15 \checkmark\end{aligned}$$

The solution checks.

45. $3\log_4 a = \log_4 27$

SOLUTION:

$$\begin{aligned}3\log_4 a &= \log_4 27\\ \log_4 a^3 &= \log_4 3^3\\ a^3 &= 3^3\\ a &= 3\end{aligned}$$

Substitute 3 for a and check the solution.

$$\begin{aligned}3\log_4 3 &= \log_4 27\\ \log_4 3^3 &= \log_4 27\\ \log_4 27 &= \log_4 27\\ 27 &= 27\checkmark\end{aligned}$$

The solution checks.

46. $2 \log_3 x + \log_3 3 = \log_3 36$

SOLUTION:

$$\begin{aligned} 2\log_3 x + \log_3 3 &= \log_3 36 \\ \log_3 x^2 + \log_3 3 &= \log_3 36 \\ \log_3 3x^2 &= \log_3 36 \\ 3x^2 &= 36 \\ x^2 &= 12 \\ x &= 2\sqrt{3} \end{aligned}$$

Substitute $2\sqrt{3}$ for *x* and check the solution.

$$\begin{aligned} 2\log_3 2\sqrt{3} + \log_3 3 &= \log_3 36 \\ \log_3 \left(2\sqrt{3}\right)^2 + \log_3 3 &= \log_3 36 \\ \log_3 12 + \log_3 3 &= \log_3 36 \\ \log_3 36 &= \log_3 36 \\ 36 &= 36 \checkmark \end{aligned}$$

The solution checks.

47. $\log_4 n + \log_4 (n - 4) = \log_4 5$

SOLUTION:

$$\begin{aligned} \log_4 n + \log_4 (n-4) &= \log_4 5 \\ \log_4 \left(n(n-4)\right) &= \log_4 5 \\ \log_4 \left(n^2 - 4n\right) &= \log_4 5 \\ n^2 - 4n &= 5 \\ n^2 - 4n - 5 &= 0 \\ (n-5)(n+1) &= 0 \end{aligned}$$

By zero Product property:

$$\begin{aligned} n - 5 &= 0 \text{ or } & n + 1 &= 0 \\ n &= 5 \quad \text{or} & n &= -1 \end{aligned}$$

The *x*-value –1 makes the argument negative. Logarithm is not defined for negative numbers. Therefore, the solution is 5.
Substitute 5 and –1 for *x* and check the solution.

$$\begin{aligned} \log_4 5 + \log_4 (5-4) &= \log_4 5 \\ \log_4 5 + \log_4 1 &= \log_4 5 \\ \log_4 5 + 0 &= \log_4 5 \\ \log_4 5 &= \log_4 5 \checkmark \\ \log_4 (-1) + \log_4 (-1-4) &= \log_4 5 \\ \log_4 -1 + \log_4 -5 &= \log_4 5 \times \end{aligned}$$

Therefore, the solution is 5.

48. **SOUND** Use the formula $L = 10 \log_{10} R$, where L is the loudness of a sound and R is the sound's relative intensity, to find out how much louder 20 people talking would be than one person talking. Suppose the sound of one person talking has a relative intensity of 80 decibels.

SOLUTION:

Substitute 80 for R and solve for L.

$$\begin{aligned} L &= 10\log_{10} 80 \\ &\approx 19.03090 \end{aligned}$$

If 20 people are talking at a time, the relative intensity of the sound is

$$20 \times 19.031 \approx 380.62$$

Subtract the loudness of one person talking.

$$380.62 - 19.031 \approx 361.6$$

Solve each equation or inequality. Round to the nearest ten-thousandth.

49. $3^x = 15$

SOLUTION:

$$\begin{aligned} 3^x &= 15 \\ \log 3^x &= \log 15 \\ x\log 3 &= \log 15 \\ x &= \frac{\log 15}{\log 3} \\ &\approx 2.4650 \end{aligned}$$

50. $6^{x^2} = 28$

SOLUTION:

$$\begin{aligned} 6^{x^2} &= 28 \\ \log 6^{x^2} &= \log 28 \\ x^2\log 6 &= \log 28 \\ x^2 &= \frac{\log 28}{\log 6} \\ x &= \sqrt{\frac{\log 28}{\log 6}} \\ &\approx \pm 1.3637 \end{aligned}$$

51. $8^{m+1} = 30$

SOLUTION:

$$\begin{aligned} 8^{m+1} &= 30 \\ \log 8^{m+1} &= \log 30 \\ (m+1)\log 8 &= \log 30 \\ m+1 &= \frac{\log 30}{\log 8} \\ m &= \frac{\log 30}{\log 8} - 1 \\ &\approx 0.6356 \end{aligned}$$

52. $12^{r-1} = 7^r$

SOLUTION:

$$\begin{aligned} 12^{r-1} &= 7^r \\ \log 12^{r-1} &= \log 7^r \\ (r-1)\log 12 &= r\log 7 \\ \frac{r-1}{r} &= \frac{\log 7}{\log 12} \\ 1 - \frac{1}{r} &= \frac{\log 7}{\log 12} \\ \frac{1}{r} &= 1 - \frac{\log 7}{\log 12} \\ &= \frac{\log 12 - \log 7}{\log 12} \\ r &= \frac{\log 12}{\log 12 - \log 7} \\ &\approx 4.6102 \end{aligned}$$

53. $3^{5n} > 24$

SOLUTION:

$$\begin{aligned} 3^{5n} &> 24 \\ \log 3^{5n} &> \log 24 \\ 5n\log 3 &> \log 24 \\ 5n &> \frac{\log 24}{\log 3} \\ n &> \frac{1}{5} \cdot \frac{\log 24}{\log 3} \\ n &> 0.5786 \end{aligned}$$

54. $5^{x+2} \le 3^x$

SOLUTION:

$$\begin{aligned} 5^{x+2} &\le 3^x \\ \log 5^{x+2} &\le \log 3^x \\ (x+2)\log 5 &\le x\log 3 \\ \frac{x+2}{x} &\le \frac{\log 3}{\log 5} \\ 1+\frac{2}{x} &\le \frac{\log 3}{\log 5} \\ \frac{2}{x} &\le \frac{\log 3}{\log 5}-1 \\ \frac{2}{x} &\le \frac{\log 3-\log 5}{\log 5} \\ x &\le \frac{2\log 5}{\log 3-\log 5} \\ &\le -6.3013 \end{aligned}$$

55. **SAVINGS** You deposited \$1000 into an account that pays an annual interest rate r of 5% compounded quarterly. Use $A=P\left(1+\frac{r}{n}\right)^{nt}$.

a. How long will it take until you have \$1500 in your account?
b. How long it will take for your money to double?

SOLUTION:

a. Substitute 1500, 1000, 0.05 and 4 for A, P, r and n then solve for t.

$$\begin{aligned} 1500 &= 1000\left(1+\frac{0.05}{4}\right)^{4t} \\ 1.5 &= \left(1+\frac{0.05}{4}\right)^{4t} \\ 1.5 &= 1.0125^{4t} \\ \log 1.5 &= \log 1.0125^{4t} \\ 4t\log 1.0125 &= \log 1.5 \\ t &= \frac{1}{4}\cdot\frac{\log 1.5}{\log 1.0125} \\ &\approx 8.2 \end{aligned}$$

It will take about 8.2 years.

b. Substitute 2000, 1000, 0.05 and 4 for A, P, r and n then solve for t.

$$\begin{aligned} 2000 &= 1000\left(1+\frac{0.05}{4}\right)^{4t} \\ 2 &= \left(1+\frac{0.05}{4}\right)^{4t} \\ 2 &= 1.0125^{4t} \\ \log 2 &= \log 1.0125^{4t} \\ 4t\log 1.0125 &= \log 2 \\ t &= \frac{1}{4}\cdot\frac{\log 2}{\log 1.0125} \\ &\approx 13.9 \end{aligned}$$

It will take about 13.4 years.

Solve each equation or inequality. Round to the nearest ten-thousandth.

56. $4e^x - 11 = 17$

SOLUTION:

$$\begin{aligned} 4e^x - 11 &= 17 \\ 4e^x &= 28 \\ e^x &= 7 \\ \ln e^x &= \ln 7 \\ x &= \ln 7 \\ &\approx 1.9459 \end{aligned}$$

57. $2e^{-x} + 1 = 15$

SOLUTION:

$$\begin{aligned} 2e^{-x} + 1 &= 15 \\ 2e^{-x} &= 14 \\ e^{-x} &= 7 \\ \ln e^{-x} &= \ln 7 \\ -x &= \ln 7 \\ x &= -\ln 7 \\ x &\approx -1.9459 \end{aligned}$$

58. $\ln 2x = 6$

SOLUTION:

$$\begin{aligned} \ln 2x &= 6 \\ 2x &= e^6 \\ x &= \frac{e^6}{2} \\ &\approx 201.7144 \end{aligned}$$

59. $2 + e^x > 9$

SOLUTION:

$$\begin{aligned} 2 + e^x &> 9 \\ e^x &> 7 \\ \ln e^x &> \ln 7 \\ x &> 1.9459 \end{aligned}$$

60. $\ln (x + 3)^5 < 5$

SOLUTION:

$$\begin{aligned} \ln(x+3)^5 &< 5 \\ 5\ln(x+3) &< 5 \\ \ln(x+3) &< 1 \\ x+3 &< e^1 \\ x &< e^1 - 3 \\ x &< -0.2817 \end{aligned}$$

61. $e^{-x} > 18$

SOLUTION:

$$\begin{aligned} e^{-x} &> 18 \\ \ln e^{-x} &> \ln 18 \\ -x &> \ln 18 \\ x &> -\ln 18 \\ x &> -2.8904 \end{aligned}$$

62. **SAVINGS** If you deposit $2000 in an account paying 6.4% interest compounded continuously, how long will it take for your money to triple? Use $A = Pe^{rt}$.

SOLUTION:

Substitute 6000, 2000 and 0.064 for A, P and r in the equation $A = Pe^{rt}$ then solve for t.

$$\begin{aligned} 6000 &= 2000e^{0.064t} \\ 3 &= e^{0.064t} \\ \ln 3 &= \ln e^{0.064t} \\ 0.064t &= \ln 3 \\ t &= \frac{\ln 3}{0.064} \\ &\approx 17.2 \end{aligned}$$

It will take about 17.2 years.

63. **CARS** Abe bought a used car for $2500. It is expected to depreciate at a rate of 25% per year. What will be the value of the car in 3 years?

SOLUTION:

Substitute 2500, 3 and 0.25 for a, t and r in the equation $y = a(1-r)^t$ then evaluate.

$$\begin{aligned} y &= a(1-r)^t \\ y &= 2500(1-0.25)^3 \\ &= 2500(0.75)^3 \\ &\approx 1054.69 \end{aligned}$$

The value of the car will be about $1054.69.

64. **BIOLOGY** For a certain strain of bacteria, k is 0.728 when t is measured in days. Using the formula $y = ae^{kt}$, how long will it take 10 bacteria to increase to 675 bacteria?

SOLUTION:

Substitute 0.728, 10 and 675 for k, a and y in the equation $y = ae^{kt}$ then solve for t.

$$\begin{aligned} y &= ae^{kt} \\ 675 &= 10e^{0.728t} \\ 67.5 &= e^{0.728t} \\ \ln 67.5 &= \ln e^{0.728t} \\ 0.728t &= \ln 67.5 \\ t &= \frac{\ln 67.5}{0.728} \\ &\approx 5.8 \end{aligned}$$

It will take about 5.8 days.

65. **POPULATION** The population of a city 20 years ago was 24,330. Since then, the population has increased at a steady rate each year. If the population is currently 55,250, find the annual rate of growth for this city.

SOLUTION:

Substitute 24330, 20 and 55250 for y, t and a in the equation $y = ae^{kt}$ then solve for k.

$$\begin{aligned} y &= ae^{kt} \\ 55250 &= 24330e^{k20} \\ e^{k20} &= \frac{55250}{24330} \\ 20k &= \ln\left(\frac{55250}{24330}\right) \\ k &= \frac{\ln\left(\frac{55250}{24330}\right)}{20} \\ &\approx 0.041 \end{aligned}$$

The annual rate of growth for this city is about 0.041 or about 4.1%.