Choose a term from the list above that best completes each statement or phrase.

1. A(n) _______ is a rational expression whose numerator and/or denominator contains a rational expression.

SOLUTION:
complex fraction

2. If two quantities show _______, their product is equal to a constant k.

SOLUTION:
inverse variation

3. A(n) _______ asymptote is a linear asymptote that is neither horizontal nor vertical.

SOLUTION:
oblique

4. A(n) _______ can be expressed in the form $y = kx$.

SOLUTION:
direct variation

5. Equations that contain one or more rational expressions are called _______.

SOLUTION:
rational equations

6. The graph of $y = \frac{x}{x+2}$ has a(n) _______ at $x = -2$.

SOLUTION:
vertical asymptote

7. _______ occurs when one quantity varies directly as the product of two or more other quantities.

SOLUTION:
Joint variation

8. A ratio of two polynomial expressions is called a(n) _______.

SOLUTION:
rational expression

9. _______ occurs when one quantity varies directly and/or inversely as two or more other quantities.

SOLUTION:
Combined variation

10. _______ looks like a hole in a graph because the graph is undefined at that point.

SOLUTION:
Point discontinuity

Simplify each expression.

11. $\frac{-16xy}{27z} \cdot \frac{15z^3}{8x^2}$

SOLUTION:

$$\frac{-16xy}{27z} \cdot \frac{15z^3}{8x^2} = \frac{(-16)(15)xyz^3}{(27)(8)x^2z}$$
$$= \frac{(-2)(5)yz^2}{(9)x}$$
$$= -\frac{10yz^2}{9x}$$

12. $\frac{x^2-2x-8}{x^2+x-12} \cdot \frac{x^2+2x-15}{x^2+7x+10}$

SOLUTION:

$$\frac{x^2-2x-8}{x^2+x-12} \cdot \frac{x^2+2x-15}{x^2+7x+10}$$
$$= \frac{(x-4)(x+2)}{(x+4)(x-3)} \cdot \frac{(x+5)(x-3)}{(x+5)(x+2)}$$
$$= \frac{x-4}{x+4}$$

13. $\frac{x^2-1}{x^2-4} \cdot \frac{x^2-5x-14}{x^2-6x-7}$

SOLUTION:

$$\frac{x^2-1}{x^2-4} \cdot \frac{x^2-5x-14}{x^2-6x-7}$$
$$= \frac{(x+1)(x-1)}{(x+2)(x-2)} \cdot \frac{(x-7)(x+2)}{(x-7)(x+1)}$$
$$= \frac{x-1}{x-2}$$

14. $\frac{x+y}{15x} \div \frac{x^2-y^2}{3x^2}$

SOLUTION:

$$\frac{x+y}{15x} \div \frac{x^2-y^2}{3x^2} = \frac{x+y}{15x} \cdot \frac{3x^2}{x^2-y^2}$$
$$= \frac{x+y}{15x} \cdot \frac{3x^2}{(x-y)(x+y)}$$
$$= \frac{x}{5(x-y)}$$

15. $\frac{\frac{x^2+3x-18}{x+4}}{\frac{x^2+7x+6}{x+4}}$

SOLUTION:

$$\frac{\frac{x^2+3x-18}{x+4}}{\frac{x^2+7x+6}{x+4}}$$
$$= \frac{x^2+3x-18}{x+4} \cdot \frac{x+4}{x^2+7x+6}$$
$$= \frac{(x+6)(x-3)}{x+4} \cdot \frac{x+4}{(x+6)(x+1)}$$
$$= \frac{x-3}{x+1}$$

16. **GEOMETRY** A triangle has an area of $3x^2 + 9x - 54$ square centimeters. If the height of the triangle is $x + 6$ centimeters, find the length of the base.

SOLUTION:

Let b = length of the base of the triangle.

$$3x^2 + 9x - 54 = \frac{1}{2}b(x+6)$$
$$\frac{3(x^2+3x-18)}{x+6} = \frac{1}{2}b$$
$$\frac{6(x+6)(x-3)}{(x+6)} = b$$
$$6(x-3) = b$$
$$6x - 18 = b$$

The length of the base of the triangle is $6x - 18$ centimeters.

Simplify each expression.

17. $\frac{9}{4ab} + \frac{5a}{6b^2}$

SOLUTION:

$$\frac{9}{4ab} + \frac{5a}{6b^2} = \frac{9}{4ab}\left(\frac{3b}{3b}\right) + \frac{5a}{6b^2}\left(\frac{2a}{2a}\right)$$
$$= \frac{27b}{12ab^2} + \frac{10a^2}{12ab^2}$$
$$= \frac{27b + 10a^2}{12ab^2}$$

18. $\frac{3}{4x-8} - \frac{x-1}{x^2-4}$

SOLUTION:

$$\frac{3}{4x-8} - \frac{x-1}{x^2-4}$$
$$= \frac{3}{4(x-2)} - \frac{x-1}{(x+2)(x-2)}$$
$$= \frac{3}{4(x-2)}\frac{(x+2)}{(x+2)} - \frac{x-1}{(x+2)(x-2)} \cdot \frac{4}{4}$$
$$= \frac{3x+6}{4(x-2)(x+2)} - \frac{4(x-1)}{4(x-2)(x+2)}$$
$$= \frac{3x+6-4x+4}{4(x-2)(x+2)}$$
$$= \frac{-x+10}{4(x-2)(x+2)}$$

19. $\frac{y}{2x} + \frac{4y}{3x^2} - \frac{5}{6xy^2}$

SOLUTION:

$$\frac{y}{2x} + \frac{4y}{3x^2} - \frac{5}{6xy^2}$$
$$= \frac{y}{2x} \cdot \frac{3xy^2}{3xy^2} + \frac{4y}{3x^2} \cdot \frac{2y^2}{2y^2} - \frac{5}{6xy^2} \cdot \frac{x}{x}$$
$$= \frac{3xy^3}{6x^2y^2} + \frac{8y^2}{6x^2y^2} - \frac{5x}{6x^2y^2}$$
$$= \frac{3xy^3 + 8y^2 - 5x}{6x^2y^2}$$

20. $\dfrac{2}{x^2-3x-10}-\dfrac{6}{x^2-8x+15}$

SOLUTION:

$$\frac{2}{x^2-3x-10}-\frac{6}{x^2-8x+15}$$
$$=\frac{2}{(x-5)(x+2)}-\frac{6}{(x-5)(x-3)}$$
$$=\frac{2}{(x-5)(x+2)}\cdot\frac{x-3}{x-3}-\frac{6}{(x-5)(x-3)}\cdot\frac{x+2}{x+2}$$
$$=\frac{2(x-3)}{(x-5)(x+2)(x-3)}-\frac{6(x+2)}{(x-5)(x-3)(x+2)}$$
$$=\frac{2x-6-6x-12}{(x-5)(x+2)(x-3)}$$
$$=\frac{-4x-18}{(x-5)(x+2)(x-3)}$$

21. $\dfrac{3}{3x^2+2x-8}+\dfrac{4x}{2x^2+6x+4}$

SOLUTION:

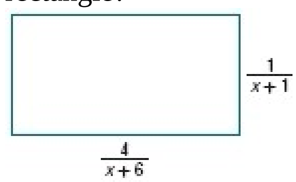
$$\frac{3}{3x^2+2x-8}+\frac{4x}{2x^2+6x+4}$$
$$=\frac{3}{(3x-4)(x+2)}+\frac{4x}{(x+2)(2x+2)}$$
$$=\frac{3(2x+2)+4x(3x-4)}{(3x-4)(x+2)(2x+2)}$$
$$=\frac{6x+6+12x^2-16x}{2(3x-4)(x+2)(x+1)}$$
$$=\frac{12x^2-10x+6}{2(x+2)(x+1)(3x-4)}$$
$$=\frac{12x^2-10x+6}{2(x+2)(x+1)(3x-4)}$$

22. $\dfrac{\dfrac{3}{2x+3}-\dfrac{x}{x+1}}{\dfrac{2x}{x+1}+\dfrac{5}{2x+3}}$

SOLUTION:

$$\frac{\dfrac{3}{2x+3}-\dfrac{x}{x+1}}{\dfrac{2x}{x+1}+\dfrac{5}{2x+3}}=\frac{\left(\dfrac{3(x+1)-x(2x+3)}{(2x+3)(x+1)}\right)}{\left(\dfrac{2x(2x+3)+5(x+1)}{(x+1)(2x+3)}\right)}$$
$$=\frac{3x+3-2x^2-3x}{4x^2+6x+5x+5}$$
$$=\frac{-2x^2+3}{4x^2+11x+5}$$

23. **GEOMETRY** What is the perimeter of the rectangle?



SOLUTION:

$$2\left(\frac{1}{x+1}\right)+2\left(\frac{4}{x+6}\right)=\frac{2}{x+1}+\frac{8}{x+6}$$
$$=\frac{2(x+6)+8(x+1)}{(x+1)(x+6)}$$
$$=\frac{2x+12+8x+8}{(x+1)(x+6)}$$
$$=\frac{10x+20}{(x+1)(x+6)}$$

Graph each function. State the domain and range.

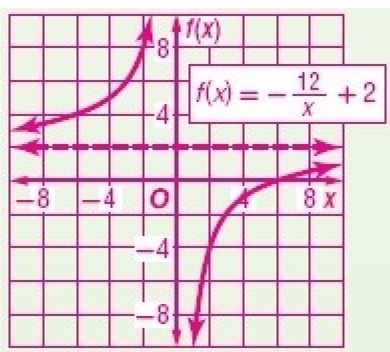
24. $f(x) = -\frac{12}{x} + 2$

SOLUTION:

The graph of $f(x) = -\frac{12}{x} + 2$ represents a transformation of the graph of $f(x) = \frac{1}{x}$.

$a = -12$: The graph is expanded and is reflected across the x-axis.

$k = 2$: The graph is translated 2 units up. There is an asymptote at $f(x) = 2$.



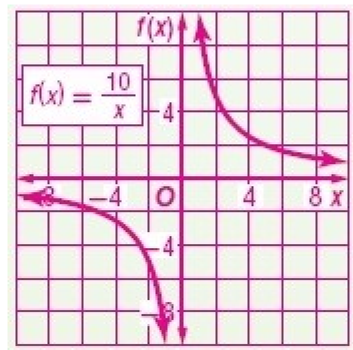
$D = \{x \mid x \neq 0\}, R = \{f(x) \mid f(x) \neq 2\}$

25. $f(x) = \frac{10}{x}$

SOLUTION:

The parent function is $f(x) = \frac{1}{x}$.

$a = 10$. The graph is expanded.



$D = \{x \mid x \neq 0\}, R = \{f(x) \mid f(x) \neq 0\}$

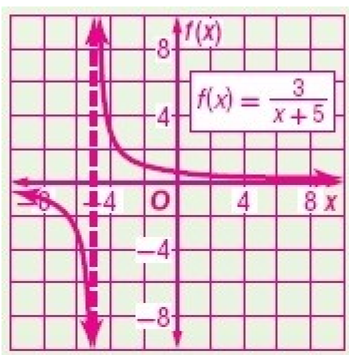
26. $f(x) = \frac{3}{x+5}$

SOLUTION:

The graph of $f(x) = \frac{3}{x+5}$ represents a transformation of the graph of $f(x) = \frac{1}{x}$.

a = 3: The graph is expanded.

h = −5: The graph is translated 5 units left. There is an asymptote at x = −5.



$D = \{x \mid x \neq -5\}$, $R = \{f(x) \mid f(x) \neq 0\}$

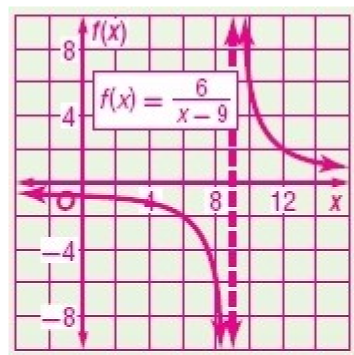
27. $f(x) = \frac{6}{x-9}$

SOLUTION:

The graph of $f(x) = \frac{6}{x-9}$ represents a transformation of the graph of $f(x) = \frac{1}{x}$.

a = 6: The graph is expanded.

h = 9: The graph is translated 9 units right. There is an asymptote at x = 9.



$D = \{x \mid x \neq 9\}$, $R = \{f(x) \mid f(x) \neq 0\}$

28. $f(x)=\frac{7}{x-2}+3$

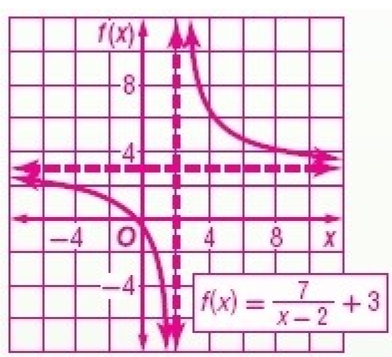
SOLUTION:

The graph of $f(x)=\frac{7}{x-2}+3$ represents a transformation of the graph of $f(x)=\frac{1}{x}$.

a = 7: The graph is expanded.

h = 2: The graph is translated 2 units right. There is an asymptote at x = 2.

k = 3: The graph is translated 3 units up. There is an asymptote at $f(x)=3$.



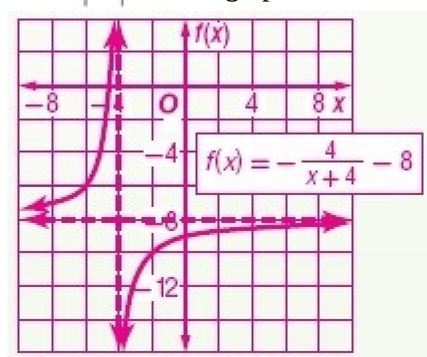
$D=\{x \mid x \neq 2\}$, $R=\{f(x) \mid f(x) \neq 3\}$.

29. $f(x)=-\frac{4}{x+4}-8$

SOLUTION:

The graph of $f(x)=-\frac{4}{x+4}-8$ represents a transformation of the graph of $f(x)=\frac{1}{x}$.

a = −4: Since a < 0, the graph is reflected across the x-axis.
h = −4: The graph is translated 4 units left. There is an asymptote at x = −4.
k = −8: The graph is translated 8 units down. There is an asymptote at $f(x)=-8$.
Since $|-4|>1$, the graph is stretched vertically.



$D=\{x \mid x \neq -4\}$, $R=\{f(x) \mid f(x) \neq -8\}$

30. **CONSERVATION** The student council is planting 28 trees for a service project. The number of trees each person plants depends on the number of student council members.

a. Write a function to represent this situation.

b. Graph the function.

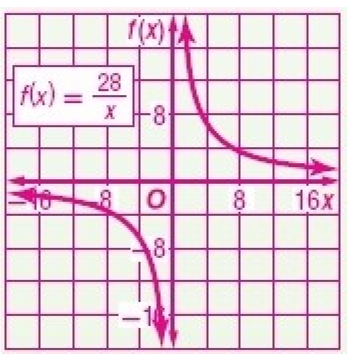
SOLUTION:

a. Let x be the number of student council members.

The function representing the situation is $f(x) = \frac{28}{x}$.

b. The graph of $f(x) = \frac{28}{x}$ represents a transformation of the graph of $f(x) = \frac{1}{x}$.

Here a = 28, the graph is stretched vertically.



Determine the equations of any vertical asymptotes and the values of *x* for any holes in the graph of each rational function.

31. $f(x) = \frac{3}{x^2 + 4x}$

SOLUTION:

$$x^2 + 4x = 0$$
$$x(x+4) = 0$$
$$x = 0 \text{ or } x = -4$$

Therefore, the vertical asymptotes are at $x = 0$ and $x = -4$.

32. $f(x) = \frac{x+2}{x^2 + 6x + 8}$

SOLUTION:

$$f(x) = \frac{x+2}{x^2+6x+8} = \frac{x+2}{(x+4)(x+2)} = \frac{1}{x+4}$$

$$x + 4 = 0$$
$$x = -4$$

Therefore, there is a vertical asymptote at $x = -4$. There is a hole at $x = -2$.

33. $f(x)=\frac{x^2-9}{x^2-5x-24}$

SOLUTION:

$$\begin{aligned}f(x)&=\frac{x^2-9}{x^2-5x-24}\\&=\frac{(x+3)(x-3)}{(x-8)(x+3)}\\&=\frac{x-3}{x-8}\end{aligned}$$

Therefore, there is a vertical asymptote at $x = 8$.
There is a holt at $x = -3$.

Graph each rational function.

34. $f(x)=\frac{x+2}{(x+5)^2}$

SOLUTION:
There is a zero at $x = -2$.

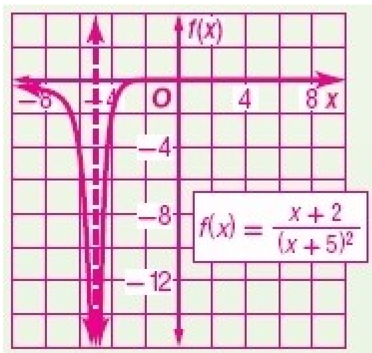
$$\begin{aligned}(x+5)^2&=0\\x+5&=0\\x&=-5\end{aligned}$$

There is a vertical asymptote at $x = -5$.

Since the degree of the numerator is less than the degree of the denominator, the horizontal asymptote is at $y = 0$.

x	$f(x)$
−7	−1.25
−6	−4
−4	−2
−3	−0.25
−2	0
−1	0.0625

Draw the asymptotes, and then use a table of values to graph the function.



35. $f(x)=\frac{x}{x+1}$

SOLUTION:

There is a zero at $x = 0$.

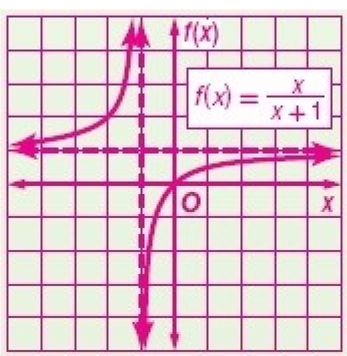
$$x+1=0$$
$$x=-1$$

There is a vertical asymptote at $x = -1$.

Since the degree of the numerator and denominator are equal, there is a horizontal asymptote at $y = 1$.

x	$f(x)$
−6	1.2
−5	1.25
−3	1.5
−2	2
0	0
1	0.5

Draw the asymptotes, and then use a table of values to graph the function.

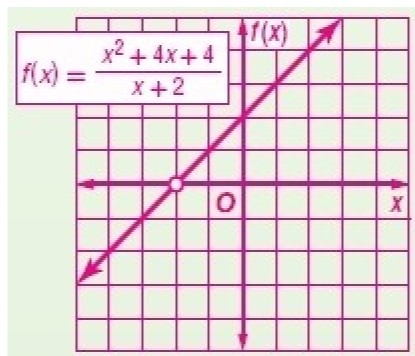


36. $f(x)=\frac{x^2+4x+4}{x+2}$

SOLUTION:

$$\begin{aligned} f(x)&=\frac{x^2+4x+4}{x+2} \\ &=\frac{(x+2)^2}{x+2} \\ &=x+2 \end{aligned}$$

The graph of $f(x)=\frac{x^2+4x+4}{x+2}$ is same as the graph of $f(x)=x+2$ with a hole at $x = -2$.



37. $f(x) = \dfrac{x-1}{x^2+5x+6}$

SOLUTION:

$f(x) = \dfrac{x-1}{x^2+5x+6}$

There is a zero at $x = 1$.

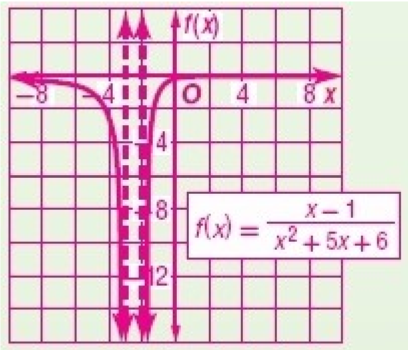
$$\begin{aligned} x^2 + 5x + 6 &= 0 \\ (x+3)(x+2) &= 0 \\ x = -3 \text{ or } x &= -2 \end{aligned}$$

The vertical asymptotes are at $x = -3$ and $x = -2$.

Since the degree of the numerator is less than the degree of the denominator, the horizontal asymptote is at $y = 0$.

x	$f(x)$
−5	−1
−4	−2.5
−1	−1
1	0
2	0.05
4	0.07143

Draw the asymptotes, and then use a table of values to graph the function.



38. **SALES** Aliyah is selling magazine subscriptions. Out of the first 15 houses, she sold subscriptions to 10 of them. Suppose Aliyah goes to x more houses and sells subscriptions to all of them. The percentage of houses that she sold to out of the total houses can be determined using

$P(x) = \dfrac{10+x}{15+x}$.

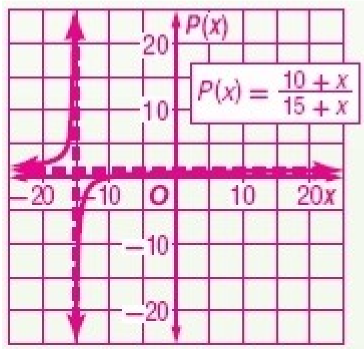
a. Graph the function.

b. What domain and range values are meaningful in the context of the problem?

SOLUTION:

a. There is a vertical asymptote at $x = -15$. Since the degree of the numerator and denominator are equal, there is a horizontal asymptote at $y = 1$.

Graph the function.



b.

$D = \{x \geq 0\}; R = \{0 \leq P(x) \leq 1.0\}$

39. If a varies directly as b and b = 18 when a = 27, find a when b = 10.

SOLUTION:

$$a = kb$$
$$27 = k(18)$$
$$k = \frac{27}{18}$$
$$k = \frac{3}{2}$$

Substitute b = 10 in the relation $a = \frac{3}{2}b$.

$$a = \frac{3}{2}(10)$$
$$a = 15$$

40. If y varies inversely as x and y = 15 when x = 3.5, find y when x = –5.

SOLUTION:

$$y = \frac{k}{x}$$
$$15 = \frac{k}{3.5}$$
$$k = 15(3.5)$$
$$k = 52.5$$

Substitute x = −5 in the relation $y = \frac{52.5}{x}$.

$$y = \frac{52.5}{-5}$$
$$= -10.5$$

41. If y varies inversely as x and y = –3 when x = 9, find y when x = 81.

SOLUTION:

$$y = \frac{k}{x}$$
$$-3 = \frac{k}{9}$$
$$k = -27$$

Substitute x = 81 in the relation $y = -\frac{27}{x}$.

$$y = -\frac{27}{x}$$
$$y = -\frac{27}{81}$$
$$= -\frac{1}{3}$$

42. If y varies jointly as x and z, and x = 8 and z = 3 when y = 72, find y when x = –2 and z = –5.

SOLUTION:

$$y = kxz$$
$$72 = k(8)(3)$$
$$72 = 24k$$
$$k = \frac{72}{24}$$
$$k = 3$$

Substitute x = −2 and z = −5 in the relation y = 3xz.

$$y = 3(-2)(-5)$$
$$= 30$$

43. If y varies jointly as x and z, and y = 18 when x = 6 and z = 15, find y when x = 12 and z = 4.

SOLUTION:

$$y = kxz$$
$$18 = k(6)(15)$$
$$18 = 90k$$
$$k = \frac{18}{90}$$
$$k = \frac{1}{5}$$

Substitute x = 12 and z = 4 in the relation $y = \frac{1}{5}xz$.

$$y = \frac{1}{5}(12)(4)$$
$$= \frac{48}{5}$$

44. **JOBS** Lisa's earnings vary directly with how many hours she babysits. If she earns \$68 for 8 hours of babysitting, find her earnings after 5 hours of babysitting.

SOLUTION:

Let x = number of hours.
Let y = Lisa's earnings.

$$y = kx$$
$$68 = k(8)$$
$$k = \frac{68}{8}$$
$$= \frac{17}{2}$$

Substitute x = 5 in the relation $y = \frac{17}{2}x$.

$$y = \frac{17}{2}x$$
$$= \frac{17}{2}(5)$$
$$= \frac{85}{2}$$
$$= 42.50$$

After 5 hours of babysitting, Lisa's earnings is \$42.50.

Solve each equation or inequality. Check your solutions.

45. $\frac{1}{3}+\frac{4}{x-2}=6$

SOLUTION:

$$\begin{aligned}\frac{1}{3}+\frac{4}{x-2}&=6\\ \frac{x-2+12}{3(x-2)}&=6\\ \frac{x+10}{3(x-2)}&=6\\ x+10&=18(x-2)\\ x+10&=18x-36\\ -17x&=-46\\ x&=\frac{46}{17}\end{aligned}$$

Check:

$$\begin{aligned}\frac{1}{3}+\frac{4}{\left(\frac{46}{17}-2\right)}&\overset{?}{=}6\\ \frac{1}{3}+\frac{4}{\left(\frac{46-34}{17}\right)}&\overset{?}{=}6\\ \frac{1}{3}+\frac{68}{12}&\overset{?}{=}6\\ \frac{4+68}{12}&\overset{?}{=}6\\ \frac{72}{12}&\overset{?}{=}6\\ 6&=6\ \checkmark\end{aligned}$$

The solution is $x=\frac{46}{17}$.

46. $\frac{6}{x+5}-\frac{3}{x-3}=\frac{6}{x^2+2x-15}$

SOLUTION:

$$\begin{aligned}\frac{6}{x+5}-\frac{3}{x-3}&=\frac{6}{x^2+2x-15}\\ \frac{6(x-3)-3(x+5)}{(x+5)(x-3)}&=\frac{6}{x^2+2x-15}\\ \frac{6x-18-3x-15}{x^2+5x-3x-15}&=\frac{6}{x^2+2x-15}\\ \frac{3x-33}{x^2+2x-15}&=\frac{6}{x^2+2x-15}\\ 3x-33&=6\\ 3x&=39\\ x&=13\end{aligned}$$

Check:

$$\begin{aligned}\frac{6}{13+5}-\frac{3}{13-3}&\overset{?}{=}\frac{6}{13^2+2(13)-15}\\ \frac{6}{18}-\frac{3}{10}&\overset{?}{=}\frac{6}{169+26-15}\\ \frac{60-54}{180}&\overset{?}{=}\frac{6}{180}\\ \frac{6}{180}&=\frac{6}{180}\ \checkmark\end{aligned}$$

The solution is x = 13.

47. $\frac{2}{x^2-9}=\frac{3}{x^2-2x-3}$

SOLUTION:

$$\begin{aligned}\frac{2}{x^2-9}&=\frac{3}{x^2-2x-3}\\ \frac{2}{(x+3)(x-3)}&=\frac{3}{(x-3)(x+1)}\\ \frac{2}{(x+3)(x-3)}\cdot\frac{(x+1)}{(x+1)}&=\frac{3}{(x-3)(x+1)}\cdot\frac{(x+3)}{(x+3)}\\ \frac{2(x+1)}{(x+3)(x-3)(x+1)}&=\frac{3(x+3)}{(x-3)(x+1)(x+3)}\\ 2x+2&=3x+9\\ -x&=7\\ x&=-7\end{aligned}$$

Check:

$$\begin{aligned}\frac{2}{(-7)^2-9}&\stackrel{?}{=}\frac{3}{(-7)^2-2(-7)-3}\\ \frac{2}{49-9}&\stackrel{?}{=}\frac{3}{49+14-3}\\ \frac{2}{40}&\stackrel{?}{=}\frac{3}{60}\\ \frac{1}{20}&=\frac{1}{20}\checkmark\end{aligned}$$

The solution is $x = -7$.

48. $\frac{4}{2x-3}+\frac{x}{x+1}=\frac{-8x}{2x^2-x-3}$

SOLUTION:

$$\begin{aligned}\frac{4}{2x-3}+\frac{x}{x+1}&=\frac{-8x}{2x^2-x-3}\\ \frac{4(x+1)+x(2x-3)}{(2x-3)(x+1)}&=\frac{-8x}{2x^2-x-3}\\ \frac{4x+4+2x^2-3x}{2x^2-x-3}&=\frac{-8x}{2x^2-x-3}\\ \frac{2x^2+x+4}{2x^2-x-3}&=\frac{-8x}{2x^2-x-3}\\ 2x^2+x+4&=-8x\\ 2x^2+9x+4&=0\end{aligned}$$

$$\begin{aligned}2x^2+9x+4&=0\\ 2x^2+x+8x+4&=0\\ x(2x+1)+4(2x+1)&=0\\ (2x+1)(x+4)&=0\\ x=-\frac{1}{2}\text{ or }x&=-4\end{aligned}$$

Check: $x=-\frac{1}{2}$

$$\begin{aligned}\frac{4}{2\left(-\frac{1}{2}\right)-3}+\frac{\left(-\frac{1}{2}\right)}{-\frac{1}{2}+1}&\stackrel{?}{=}\frac{-8\left(-\frac{1}{2}\right)}{2\left(-\frac{1}{2}\right)^2-\left(-\frac{1}{2}\right)-3}\\ \frac{4}{-4}-1&\stackrel{?}{=}\frac{4}{\left(\frac{1}{2}\right)+\frac{1}{2}-3}\\ -1-1&\stackrel{?}{=}\frac{4}{1-3}\\ -2&\stackrel{?}{=}\frac{4}{-2}\\ -2&=-2\checkmark\end{aligned}$$

Check: $x = -4$

$$\begin{aligned}\frac{4}{-8-3}+\frac{(-4)}{-4+1}&\stackrel{?}{=}\frac{-8(-4)}{2(16)-(-4)-3}\\ \frac{4}{-11}-\frac{4}{-3}&\stackrel{?}{=}\frac{32}{32+4-3}\\ -\frac{4}{11}+\frac{4}{3}&\stackrel{?}{=}\frac{32}{33}\\ \frac{-12+44}{33}&\stackrel{?}{=}\frac{32}{33}\\ \frac{32}{33}&=\frac{32}{33}\checkmark\end{aligned}$$

The solution is $x=-\frac{1}{2},-4$.

49. $\frac{x}{x+4} - \frac{28}{x^2+x-12} = \frac{1}{x-3}$

SOLUTION:

$$\begin{aligned}
\frac{x}{x+4} - \frac{28}{x^2+x-12} &= \frac{1}{x-3} \\
\frac{x}{x+4} - \frac{28}{(x+4)(x-3)} &= \frac{1}{x-3} \\
\frac{x(x-3)-28}{(x+4)(x-3)} &= \frac{1}{x-3} \\
\frac{x^2-3x-28}{(x-3)} &= \frac{x+4}{x-3} \\
x^2-3x-28 &= x+4 \\
x^2-3x-x-28-4 &= 0 \\
x^2-4x-32 &= 0 \\
(x-8)(x+4) &= 0 \\
x = 8 \text{ or } x &= -4
\end{aligned}$$

Check: $x = 8$

$$\begin{aligned}
\frac{8}{8+4} - \frac{28}{8^2+8-12} &\stackrel{?}{=} \frac{1}{8-3} \\
\frac{8}{12} - \frac{28}{64+8-12} &\stackrel{?}{=} \frac{1}{5} \\
\frac{8}{12} - \frac{28}{60} &\stackrel{?}{=} \frac{1}{5} \\
\frac{8(5)-28}{60} &\stackrel{?}{=} \frac{1}{5} \\
\frac{40-28}{60} &\stackrel{?}{=} \frac{1}{5} \\
\frac{12}{60} &\stackrel{?}{=} \frac{1}{5} \\
\frac{1}{5} &= \frac{1}{5} \checkmark
\end{aligned}$$

When $x = -4$, left side expression of the rational equation becomes undefined. Therefore, the solution of the equation is $x = 8$.

50. $\frac{x}{2} + \frac{1}{x-1} < \frac{x}{4}$

SOLUTION:

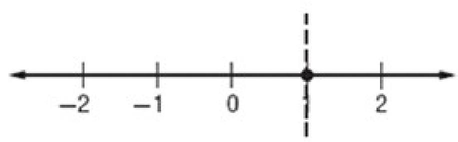
The excluded value for this inequality is $x = 1$.

Solve the related equation $\frac{x}{2} + \frac{1}{x-1} = \frac{x}{4}$.

$$\begin{aligned}
\frac{x}{2} + \frac{1}{x-1} &= \frac{x}{4} \\
\frac{x(x-1)+2}{2(x-1)} &= \frac{x}{4} \\
\frac{x^2-x+2}{2x-2} &= \frac{x}{4} \\
4(x^2-x+2) &= x(2x-2) \\
4x^2-4x+8 &= 2x^2-2x \\
2x^2-2x+8 &= 0 \\
x^2-x+4 &= 0
\end{aligned}$$

There exists no real solution for the quadratic equation $x^2 - x + 4 = 0$.

Divide the real line in to two regions as shown.



Test $x = 0$.

$$\begin{aligned}
\frac{0}{2} + \frac{1}{0-1} &\stackrel{?}{<} \frac{0}{4} \\
0-1 &\stackrel{?}{<} 0 \\
-1 &< 0 \checkmark
\end{aligned}$$

Test $x = 2$.

$$\begin{aligned}
\frac{2}{2} + \frac{1}{2-1} &\stackrel{?}{<} \frac{2}{4} \\
1+1 &\stackrel{?}{<} \frac{1}{2} \\
2 &\not< \frac{1}{2}
\end{aligned}$$

Therefore, the solution is $x < 1$.

51. $\frac{1}{2x} - \frac{4}{5x} > \frac{1}{3}$

SOLUTION:

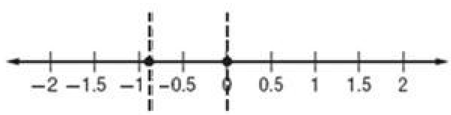
The excluded value for this inequality is $x = 0$.

Solve the related equation $\frac{1}{2x} - \frac{4}{5x} = \frac{1}{3}$.

The LCD is $30x$.

$$\begin{aligned} \frac{1}{2x} - \frac{4}{5x} &= \frac{1}{3} \\ 30x \cdot \frac{1}{2x} - 30x \cdot \frac{4}{5x} &= 30x \cdot \frac{1}{3} \\ 15 - 24 &= 10x \\ -9 &= 10x \\ -\frac{9}{10} &= x \end{aligned}$$

Draw vertical lines at the excluded value and at the solution to separate the number line into intervals.



Now test a sample value in each interval to determine whether the values in the interval satisfy the inequality.

Test $x = -1$.

$$\begin{aligned} \frac{1}{2(-1)} - \frac{4}{5(-1)} &\overset{?}{>} \frac{1}{3} \\ -\frac{1}{2} + \frac{4}{5} &\overset{?}{>} \frac{1}{3} \\ \frac{3}{10} &\overset{?}{>} \frac{1}{3} \\ \frac{9}{30} &\not> \frac{1}{3} \;✗ \end{aligned}$$

Test $x = -0.5$.

$$\begin{aligned} \frac{1}{2(-0.5)} - \frac{4}{5(-0.5)} &\overset{?}{>} \frac{1}{3} \\ -1 + \frac{8}{5} &\overset{?}{>} \frac{1}{3} \\ \frac{3}{5} &\overset{?}{>} \frac{1}{3} \\ \frac{9}{15} &> \frac{5}{15} \;\checkmark \end{aligned}$$

Test $x = 1$.

$$\begin{aligned} \frac{1}{2(1)} - \frac{4}{5(1)} &\overset{?}{>} \frac{1}{3} \\ \frac{1}{2} - \frac{4}{5} &\overset{?}{>} \frac{1}{3} \\ -\frac{3}{10} &\not> \frac{1}{3} \;✗ \end{aligned}$$

The statement is only true for $x = -0.5$. Therefore, the solution is $-\frac{9}{10} < x < 0$.

52. **YARD WORK** Lana can plant a garden in 3 hours. Milo can plant the same garden in 4 hours. How long will it take them if they work together?

SOLUTION:

$$\begin{aligned} \frac{x}{3} + \frac{x}{4} &= 1 \\ \frac{4x + 3x}{12} &= 1 \\ 7x &= 12 \\ x &= \frac{12}{7} \\ x &= 1\frac{5}{7} \end{aligned}$$

Therefore, it will take $1\frac{5}{7}$ hr to plant the garden if they work together.