Study Guide and Review - Chapter 9

State whether each sentence is *true* or *false* . If *false,* replace the underlined term to make a true sentence.

1. The set of all points in a plane that are equidistant from a given point in the plane, called the focus, forms a circle.

   ANSWER:
   false, center

2. A(n) ellipse is the set of all points in a plane such that the sum of the distances from the two fixed points is constant.

   ANSWER:
   true

3. The endpoints of the major axis of an ellipse are the foci of the ellipse.

   ANSWER:
   false, vertices

4. The radius is the distance from the center of a circle to any point on the circle.

   ANSWER:
   true

5. The line segment with endpoints on a parabola, through the focus of the parabola, and perpendicular to the axis of symmetry is called the latus rectum.

   ANSWER:
   true

6. Every hyperbola has two axes of symmetry, the transverse axis and the major axis.

   ANSWER:
   false, conjugate axis

7. A directrix is the set of all points in a plane that are equidistant from a given point in the plane, called the center

   ANSWER:
   false, circle

8. A hyperbola is the set of all points in a plane such that the absolute value of the sum of the distances from any point on the hyperbola to two given points is constant.

   ANSWER:
   false; difference

9. A parabola can be defined as the set of all points in a plane that are the same distance from the focus and a given line called the directrix.

   ANSWER:
   true

10. The major axis is the longer of the two axes of symmetry of an ellipse.

    ANSWER:
    true

Find the midpoint of the line segment with endpoints at the given coordinates.

11. (–8, 6), (3, 4)

    ANSWER:
    $\left(-\frac{5}{2}, 5\right)$

12. (–6, 0), (–1, 4)

    ANSWER:
    $\left(-\frac{7}{2}, 2\right)$

13. $\left(\frac{3}{4}, \frac{2}{3}\right), \left(-\frac{1}{3}, \frac{1}{4}\right)$

    ANSWER:
    $\left(\frac{5}{24}, \frac{11}{24}\right)$

14. (15, 20), (18, 21)

    ANSWER:
    $\left(\frac{33}{2}, \frac{41}{2}\right)$

Find the distance between each pair of points with the given coordinates.

15. (10, –3), (1, –5)

ANSWER:

$\sqrt{85}$

16. (0, 6), (–9, 7)

ANSWER:

$\sqrt{82}$

17. $\left(\frac{1}{4}, \frac{1}{2}\right), \left(\frac{3}{2}, \frac{5}{4}\right)$

ANSWER:

$\frac{\sqrt{34}}{4}$

18. (5, –3), (7, –1)

ANSWER:

$2\sqrt{2}$

19. **HIKING** Marc wants to hike from his camp to a waterfall. The waterfall is 5 miles south and 8 miles east of his campsite.

a. How far away is the waterfall?

b. Marc wants to stop for lunch halfway to the waterfall. Where should he stop?

ANSWER:

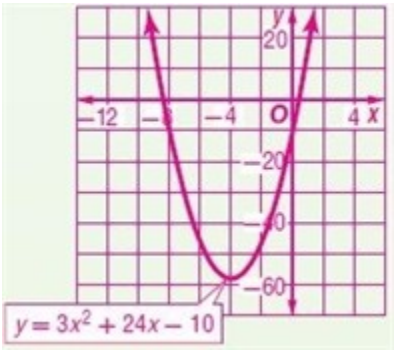
a. $\sqrt{89} \approx 9.4$ miles

b. $\left(4, -\frac{5}{2}\right)$

Graph each equation.

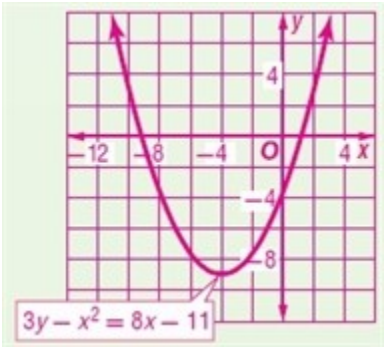
20. $y = 3x^2 + 24x - 10$

ANSWER:



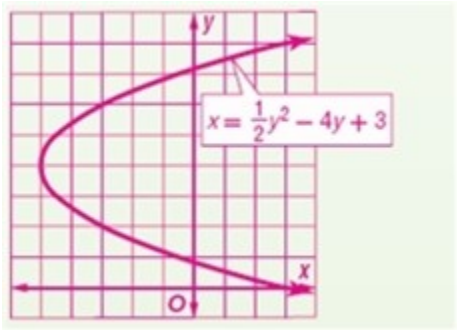
21. $3y - x^2 = 8x - 11$

ANSWER:



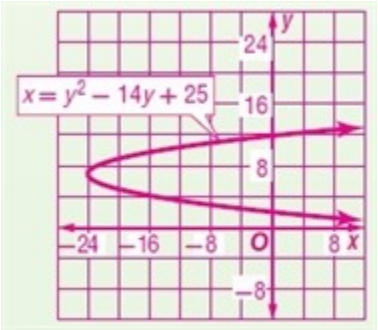
22. $x = \frac{1}{2}y^2 - 4y + 3$

ANSWER:



23. $x = y^2 - 14y + 25$

ANSWER:



Write each equation in standard form. Identify the vertex, axis of symmetry, and direction of opening of the parabola.

24. $y = \frac{1}{2}x^2$

ANSWER:

$y = -\frac{1}{2}x^2$; vertex: (0, 0); axis of symmetry: $x = 0$; opens: upward

25. $y = 4x^2 - 16x + 9$

ANSWER:

$y = 4(x-2)^2 - 7$; vertex: (2, –7); axis of symmetry: $x = 2$; opens: upward

26. $x - 6y = y^2 + 4$

ANSWER:

$x = (y+3)^2 - 5$; vertex: (–5, –3); axis of symmetry: $y = -3$; opens to the right

27. $x = y^2 + 14y + 20$

ANSWER:

$x = (y+7)^2 - 29$; vertex = (–29, –7); axis of symmetry: $y = -7$; opens to the right

28. **SPORTS** When a football is kicked, the path it travels is shaped like a parabola. Suppose a football is kicked from ground level, reaches a maximum height of 50 feet, and lands 200 feet away. Assuming the football was kicked at the origin, write an equation of the parabola that models the flight of the football.

ANSWER:

$y = -\frac{1}{200}(x-100)^2 + 50$

Write an equation for the circle that satisfies each set of conditions.

29. center (–1, 6), radius 3 units

ANSWER:

$(x+1)^2 + (y-6)^2 = 9$

30. endpoints of a diameter (2, 5) and (0, 0)

ANSWER:

$(x-1)^2 + (y-2.5)^2 = \frac{29}{4}$

31. endpoints of a diameter (4, –2) and (–2, –6)

ANSWER:

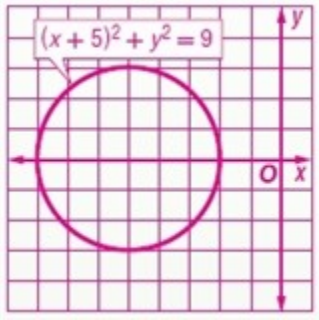
$(x-1)^2 + (y+4)^2 = 13$

Find the center and radius of each circle. Then graph the circle.

32. $(x+5)^2 + y^2 = 9$

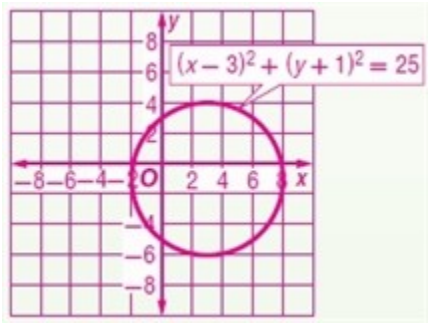
ANSWER:

(–5, 0); $r = 3$



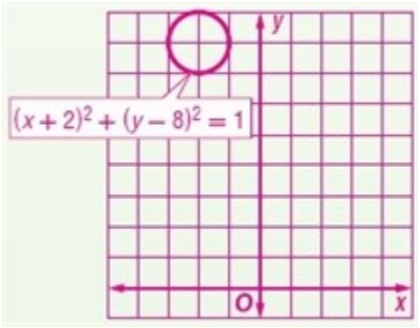
33. $(x-3)^2+(y+1)^2=25$

ANSWER:
(3, –1); $r = 5$



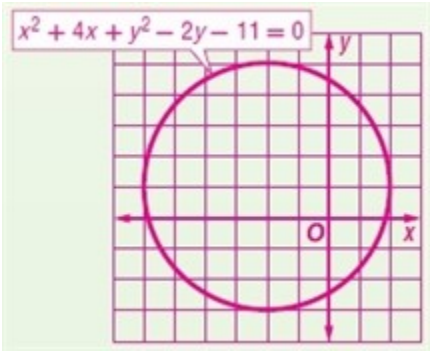
34. $(x+2)^2+(y-8)^2=1$

ANSWER:
(–2, 8); $r = 1$



35. $x^2+4x+y^2-2y-11=0$

ANSWER:
(–2, 1); $r = 4$



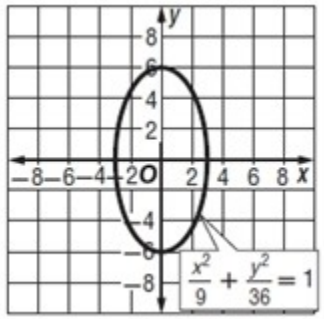
36. **SOUND** A loudspeaker in a school is located at the point (65, 40). The speaker can be heard in a circle with a radius of 100 feet. Write an equation to represent the possible boundary of the loudspeaker sound.

ANSWER:
$(x-65)^2+(y-40)^2=100^2$

Find the coordinates of the center and foci and the lengths of the major and minor axes for the ellipse with the given equation. Then graph the ellipse.

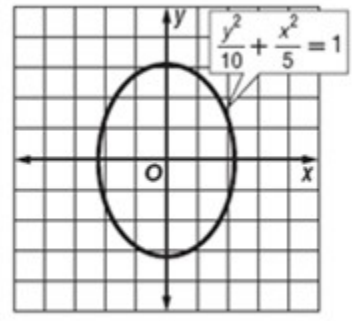
37. $\frac{x^2}{9}+\frac{y^2}{36}=1$

ANSWER:
$(0,0);\left(0,\pm 3\sqrt{3}\right);12;6$



38. $\frac{y^2}{10}+\frac{x^2}{5}=1$

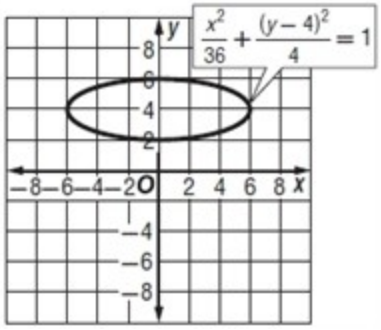
ANSWER:
$(0,0);\left(0,\pm\sqrt{5}\right);2\sqrt{10};2\sqrt{5}$



39. $\frac{x^2}{36}+\frac{(y-4)^2}{4}=1$

ANSWER:

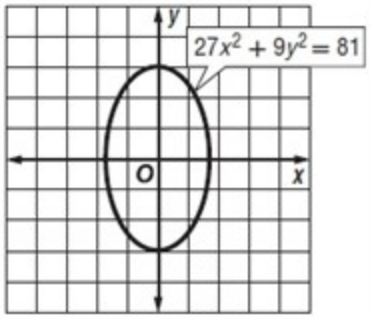
$(0,4);(\pm 4\sqrt{2},4);12;4$



40. $27x^2+9y^2=81$

ANSWER:

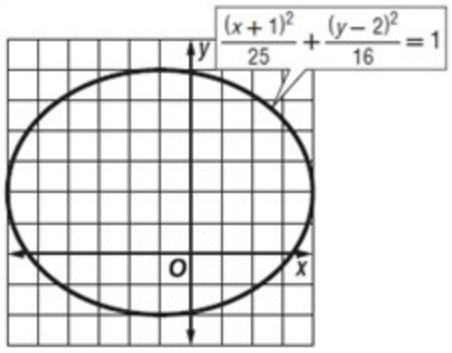
$(0,0);(0,\pm\sqrt{6});6;2\sqrt{3}$



41. $\frac{(x+1)^2}{25}+\frac{(y-2)^2}{16}=1$

ANSWER:

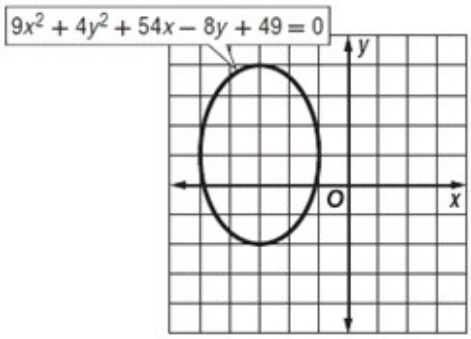
$(-1,2);(-4,2)$ and $(2,2);10;8$



42. $9x^2+4y^2+54x-8y+49=0$

ANSWER:

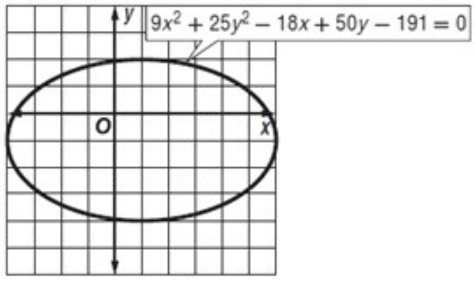
$(-3,1);(-3,1\pm\sqrt{5});6;4$



43. $9x^2+25y^2-18x+50y-191=0$

ANSWER:

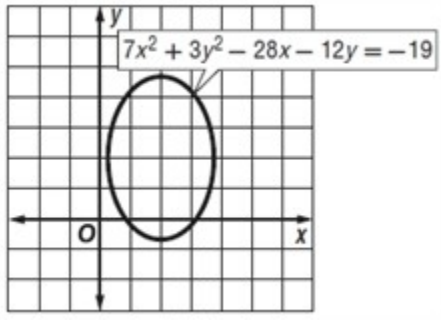
$(1,-1);(-3,-1)$ *and* $(5,-1);10;6$



44. $7x^2+3y^2-28x-12y=-19$

ANSWER:

$(2,2);(2,4);(2,0);2\sqrt{7};2\sqrt{3}$



45. **LANDSCAPING** The Martins have a garden in their front yard that is shaped like an ellipse. The major axis is 16 feet and the minor axis is 10 feet. Write an equation to model the garden. Assume the origin is at the center of the garden.

ANSWER:

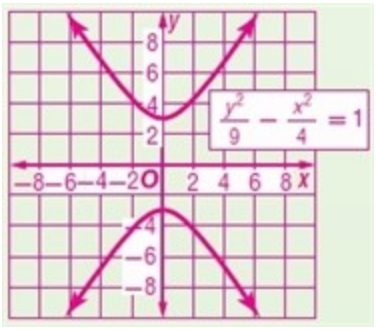
$$\frac{x^2}{64}+\frac{y^2}{25}=1$$

Graph each hyperbola. Identify the vertices, foci, and asymptotes.

46. $\frac{y^2}{9}-\frac{x^2}{4}=1$

ANSWER:

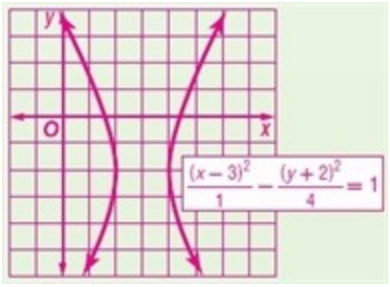
$$(0,\pm 3);\left(0,\pm\sqrt{13}\right);y=\pm\frac{3}{2}x$$



47. $\frac{(x-3)^2}{1}-\frac{(y+2)^2}{4}=1$

ANSWER:

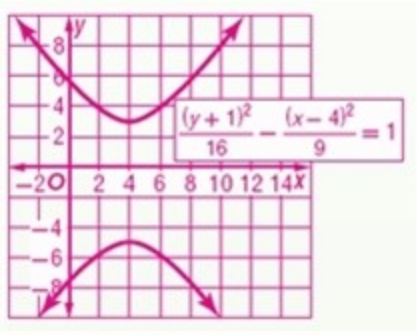
$$(2,-2);(4,-2);\left(3,\pm\sqrt{5},-2\right);y+2=\pm 2(x-3)$$



48. $\frac{(y+1)^2}{16}-\frac{(x-4)^2}{9}=1$

ANSWER:

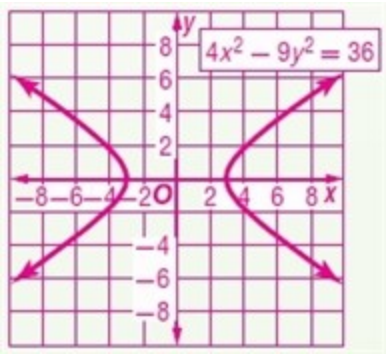
$$(4,-5);(4,3);(4,-6);(4,4);y+1=\pm\frac{4}{3}(x-4)$$



49. $4x^2-9y^2=36$

ANSWER:

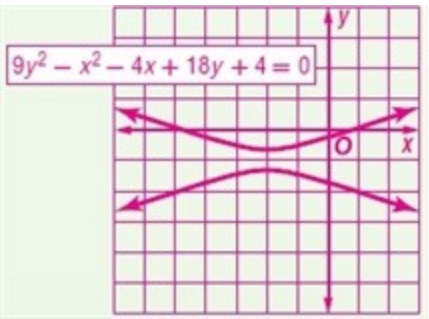
$$(\pm 3,0);\left(\pm\sqrt{13},0\right);y=\pm\frac{2}{3}x$$



50. $9y^2-x^2-4x+18y+4=0$

ANSWER:

$$\left(-2,-\frac{2}{3}\right)\left(-2,-\frac{4}{3}\right);\left(-2,-1\pm\frac{\sqrt{10}}{3}\right);y+1=\pm\frac{1}{3}(x+2)$$



51. **MIRRORS** A hyperbolic mirror is shaped like one branch of a hyperbola. It reflects light rays directed at one focus toward the other focus. Suppose a hyperbolic mirror is modeled by the upper branch of the hyperbola $\frac{y^2}{9} - \frac{x^2}{16} = 1$. A light source is located at (–10, 0). Where should the light hit the mirror so that the light will be reflected to (0, –5)?

ANSWER:

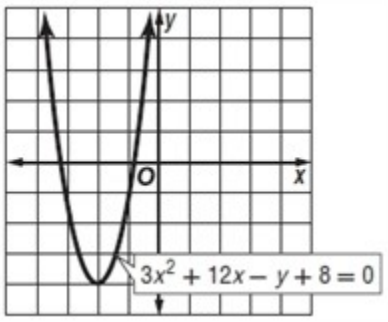
$\left(\frac{40-24\sqrt{5}}{5}, \frac{45-12\sqrt{5}}{5}\right)$

Write each equation in standard form. State whether the graph of the equation is *a parabola, circle, ellipse,* or *hyperbola*. Then graph.

52. $3x^2 + 12x - y + 8 = 0$

ANSWER:

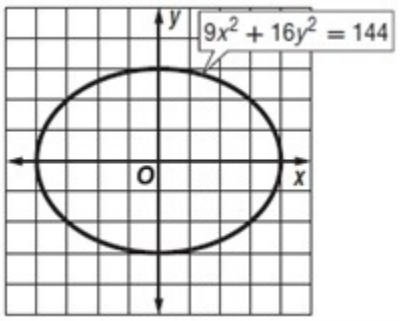
$y = 3(x+2)^2 - 4$; parabola



53. $9x^2 + 16y^2 = 144$

ANSWER:

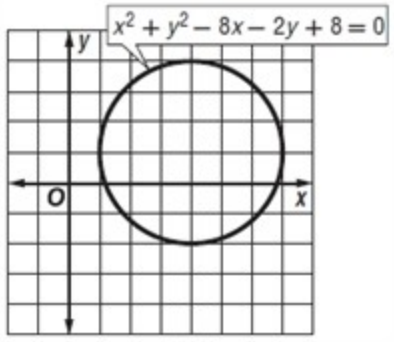
$\frac{x^2}{16} + \frac{y^2}{9} = 1$; ellipse



54. $x^2 + y^2 - 8x - 2y + 8 = 0$

ANSWER:

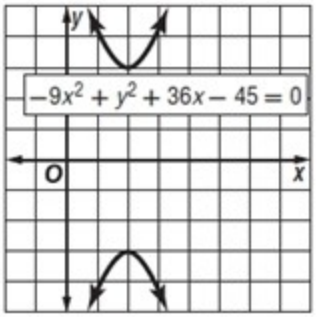
$(x-4)^2 + (y-1)^2 = 9$; circle



55. $-9x^2 + y^2 + 36x - 45 = 0$

ANSWER:

$\frac{y^2}{9} - \frac{(x-2)^2}{1} = 1$; hyperbola



Without writing the equation in standard form, state whether the graph of the equation is a *parabola, circle, ellipse,* or *hyperbola*.

56. $7x^2 + 9y^2 = 63$

ANSWER:

ellipse

57. $5y^2 + 2y + 4x - 13x^2 = 81$

ANSWER:

hyperbola

58. $x^2 - 8x + 16 = 6y$

ANSWER:

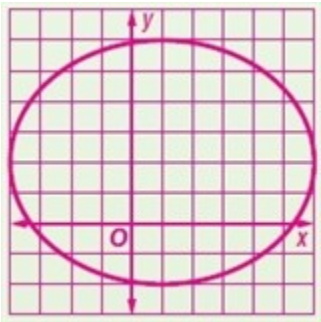
parabola

59. $x^2 + 4x + y^2 - 285 = 0$

ANSWER:
circle

60. **LIGHT** Suppose the edge of a shadow can be represented by the equation $16x^2 + 25y^2 - 32x - 100y - 284 = 0$.
a. What is the shape of the shadow?
b. Graph the equation.

ANSWER:
a. ellipse
b.



Solve each system of equations.

61. $x^2 + y^2 = 8$
$x + y = 0$

ANSWER:
(2, –2) , (–2, 2)

62. $x - 2y = 2$
$y^2 - x^2 = 2x + 4$

ANSWER:
(–2, –2)

63. $y + x^2 = 4x$
$y + 4x = 16$

ANSWER:
(4, 0)

64. $3x^2 - y^2 = 11$
$x^2 + 4y^2 = 8$

ANSWER:
(–2, –1), (–2, 1), (2, –1), (2, 1)

65. $5x^2 + y^2 = 30$
$9x^2 - y^2 = -16$

ANSWER:
$(1, \pm 5)$, $(-1, \pm 5)$

66. $\frac{x^2}{30} + \frac{y^2}{6} = 1$
$x = y$

ANSWER:
$(\sqrt{5}, \sqrt{5}), (-\sqrt{5}, -\sqrt{5})$

67. **PHYSICAL SCIENCE** Two balls are launched into the air at the same time. The heights they are launched from are different. The height y in feet of one is represented by $y = -16t^2 + 80t + 25$ where t is the time in seconds. The height of the other ball is represented by $y = -16t^2 + 30t + 100$.
a. After how many seconds are the balls at the same height?
b. What is this height?

ANSWER:
a. 1.5 seconds
b. 109 ft

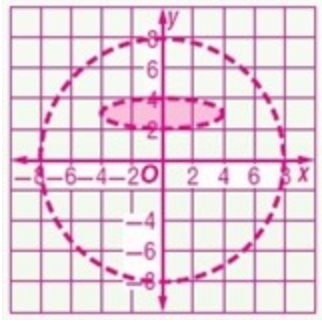
68. **ARCHITECTURE** An architect is building the front entrance of a building in the shape of a parabola with the equation $y = -\frac{1}{10}(x - 10)^2 + 20$. While the entrance is being built, the construction team puts in two support beams with equations $y = -x + 10$ and $y = x - 10$. Where do the support beams meet the parabola?

ANSWER:
(0, 10) and (20, 10)

Solve each system of inequalities by graphing.

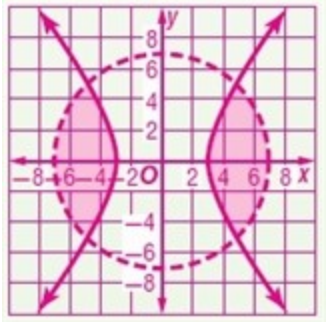
69. $x^2 + y^2 < 64$
$x^2 + 16(y-3)^2 < 16$

ANSWER:



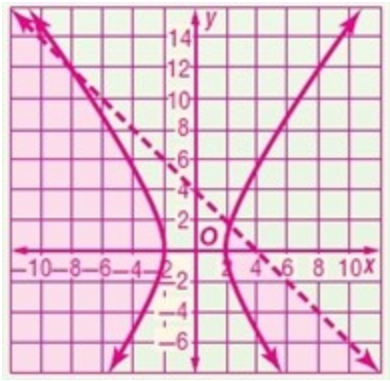
70. $x^2 + y^2 < 49$
$16x^2 - 9y^2 \geq 144$

ANSWER:



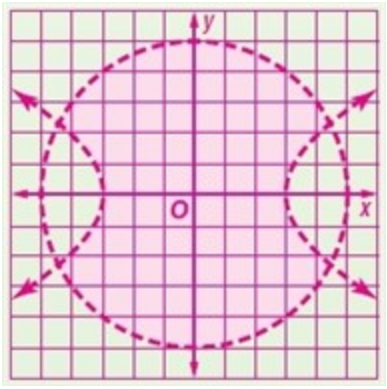
71. $x + y < 4$
$9x^2 - 4y^2 \geq 36$

ANSWER:



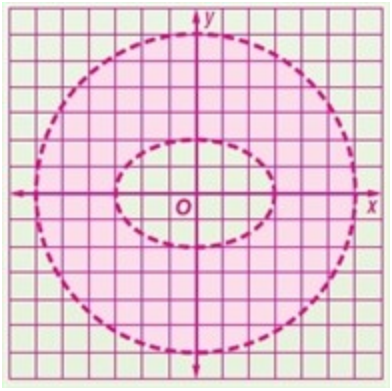
72. $x^2 + y^2 < 25$
$4x^2 - 9y^2 < 36$

ANSWER:



73. $x^2 + y^2 < 36$
$4x^2 + 9y^2 > 36$

ANSWER:



74. $y^2 < x$
$x^2 - 4y^2 < 16$

ANSWER: